Energy Efficient Power Allocation in MIMO-NOMA Systems with ZF Precoding Using Cell Division Technique

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Abstract—**In this paper, the performance of a system in terms of the energy efficiency (EE) is studied. To check the EE performance, an appropriate power is allocated to each user. The system in question in this paper is a multiple-input multiple-output (MIMO) system with non-orthogonal multiple access (NOMA) method. Precoding in this system is considered to be the zero forcing (ZF). It is also assumed that the channel state information (CSI) mode is perfect. First, all the parameters that affect the channel, such as path loss and beam forming are investigated, and then the channel matrix is obtained. To improve system performance, better conditions are provided for users with poor channel conditions. These conditions are created by allocating more appropriate power to these users, or in other words, the total transmission power is divided according to the distance of users from the base station (BS) and the channel conditions of each user. The problem of maximizing the EE is formulated with two constraints of the minimum user rate and the maximum transmission power. This is a non-convex problem that becomes a convex problem using optimization properties, and because the problem is constrained it becomes an unconstrained problem using the Lagrange dual function. Numerical and simulation results are presented to prove the mathematical relationships which show that the performance of the proposed scheme is improved compared to the existing methods. The simulation results are related to two different algorithms with a same objective function. Furthermore, to comparison with performance of other methods, output of these two algorithms are also compared with each other.**

Keywords: Multiple-Input Multiple-Output (MIMO); Energy Efficiency (EE); Power Allocation; Zero Forcing (ZF) Precoding; Non-Orthogonal Multiple Access (NOMA); Cell Division Technique.

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I. INTRODUCTION

In all tellecommunication systems where demand is increasing, there is a need for a new technology that can meet requirements of consumers. With increasing number of antennas in single-antenna systems, commonly called multiple-input multiple-output (MIMO) systems, emerged. In these systems, interference between users is reduced and rate of users is provided to reach better serving. Furthermore, by using the multiplexing created by the MIMO system, the ability to increase the reliability is achieved and it is possible to increase the spectral efficiency (SE) and the energy efficiency (EE). In order to optimize system performance and meet the requirement for a higher number of users, a multiple user MIMO (MU-MIMO) system is introduced. Due to the fact that the more antennas in the base station (BS) yeilds a greater degree of freedom, a higher number of users can communicate simultaneously in a same time and frequency resources [1]-[2].

The rapid growth of mobile internet leads to an increase in data traffic; thus, the SE is a key challenge for a sudden increase in the data traffic. Furthermore, fifth generation (5G) telecommunication networks need to support a large number of users with the lowest cost and several other services. To address all these challenges, the non-orthogonal multiple access (NOMA) has attracted attention of researchers. In the NOMA, the user's signal is sent unintentionally by the sender. This way of sending causes intracellular interference. By employing the successive interference cancellation technique on the receiver side, we can separate the user signal and eliminate interference. In other words, the NOMA has to endure receiver complexity to increase spectral gain [3].

In [4], an efficient power allocation scheme for a MIMO-NOMA system is proposed. The modeled system of this paper uses the zero forcing (ZF) beamforming method. The effects of factors such as path-loss and channel estimation error are also considered. These calculations are performed for several clusters which most of researches do not have this assumption and only examine in a single cluster. The solution method is based on the Karush-Kuhn-Tucker method. An algorithm as a numerical solution is proposed and the improvement obtained by the proposed algorithm is clearly shown in the implementation results.

The EE of a massive MIMO system is investigated using the NOMA method in [5]. The method proposed in this paper to improve system performance from the EE point of view for the system is the cell division technique. The basic premise of this idea is that users who are relatively far from the BS receive a larger amount of the total transmission power, and users who are close to the BS should receive less power. How the total power is allocated to users depends on the distance of users from the BS. By using the Lagrange dual function method, an iterative algorithm based on the standard interference function is given to solve the optimization problem numerically. The obtained results show that the cell segmentation technique significantly improves the system performance.

A discussion on the operation of a massive MIMO system is provided in [6]. By using the appropriate power allocation, an attempt has been made to maximize the EE value of the system. For this purpose, two constraints regarding to the minimum power required by users and the maximum transmission power have been assumed. The problem is divided into two cases. If the sum of the minimum power for all users is less than the maximum power of transmission, then the problem of maximizing EE arises. Otherwise, the operation of adding users to the desired cluster is raised. The method utilized to solve the problem is the KKT method. Furthermore, an iterative algorithm is presented for numerically solving of the problem. The authors also provide computer results to substantiate their claim.

In research paper [7], the EE of massive MIMO system is also investigated. In most of researches, the negative effects of eavesdropper on a telecommunications system are not considered. In this paper, the authors evaluate the performance of a telecommunication system in the presence of an eavesdropper. The eavesdropper tries to destroy the information that is communicated between the BS and the users. The precoding scheme which is employed is the maximum ratio transmit.

In this paper, we try to examine the performance of MIMO-NOMA system. Using techniques of optimization theory, the main optimization problem becomes a convex optimization problem. However, the problem cannot be solved simply, and by using the Lagrange dual function method, the optimization problem becomes an unconstraint one.

To improve system performance, we exploit the cell division technique. This means that users who are further away from the BS, or in other words have poor channel conditions, have a greater allocated power. On the other hand, users who have a shorter distance from the BS, or in other words, have stronger channel conditions, have a smaller share of the maximum transmission power. We provide results obtained from implementation and simulation to prove the efficiency of the proposed scheme. Two iterative algorithms are presented to obtain numerical results that evaluate and compare different methods.

In the continuation of this paper and in Section II, the proposed model of the system is introduced and also a definition of the EE is provided as a performance measure. In Section III, first the optimization problem is formulated, and then a strategy to solve this optimization problem is presented and practical algorithms are given to get the numerical results. In Section IV, we assess achieved improvements in the system performance with simulation and computer software results. Section V, which is the final section, discusses the conclusions and future works.

II. SYSTEM MODEL

In this section, the proposed system model is obtained in the form of mathematical relations. Then,

we calculate the channel matrix and receiver is formulated. Finally, a definition of sum data rate and the EE of system is provided. As it is seen in Figure 1, the system that we study is a MIMO system. All relations are obtained in the downlink. This system has a BS that is equipped with M transmitter antennas. Users are distributed around the BS, each of them has a single receive antenna. Furthermore, users are distributed randomly around the BS. The number of these users is considered to be K. The grouping way of these users is also shown in Figure 1, in which users at the edge of the cell are called far users who have poor channel conditions. The other category include users which are close to the BS and have strong channel conditions. In other words, if we consider the cell radius D, users whose distance is more than $D/2$ are called far users and users whose distance is less than $D/2$ are called close users.

The channel between the BS and users is assumed to be a Rayleigh fading channel. In this system model, it is assumed that the channel state information (CSI) is perfect. To formulate the ZF precoding, the channel matrix must be obtained according to the following steps by inserting the effect of important factors which play prominent role.
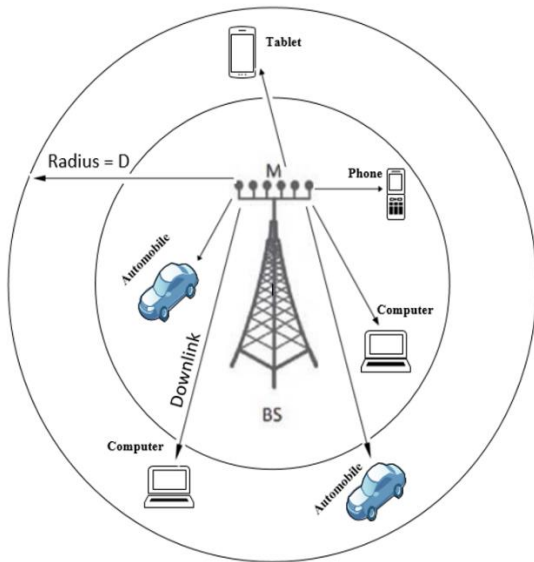

Figure 1. System model.

A. Modeling the path-loss parameter

Important factors to consider in modeling the path-loss include the distance of the users to the BS and the reference distance, the fixed path-loss parameter that depends on the propagation environment, and a fixed number called the path-loss constant. With these parameters expressed, the path loss relationship is formulated as follows,

$$l_k = J_c - 10\nu \times \log_{10}\left(\frac{d_k}{\Delta}\right), \qquad (1)$$

where, J_c represents the path-loss constant, ν is the path-loss coefficient. d_k indicates the distance

between users and the BS and also Δ is the reference distance.

B. Calculating the ideal channel matrix

As mentioned, the channel between users and the BS is considered as a Rayleigh fading. According to the number of transmitter antennas and the number of users, the telecommunication channel relationship is modeled as follows,

$$\mathbf{G} = \frac{1}{\sqrt{2}}\left[\Re(M,K) + \sqrt{-1} \times \Re(M,K)\right], \qquad (2)$$

where, $\mathcal{R}(M,K)$ generates $M \times K$ matrices with standard Gaussian random variable elements.

C. The effect of path-loss on the ideal channel matrix

Various parameters such as the distance of users from the BS and the type of propagation environment affect the telecommunication channel, all of which are included in relation of the path-loss. The effect of path-loss on the channel is applied as follows in the telecommunication channel matrix C,

$$\mathbf{C} = \mathbf{G} \times \mathbf{L}. \qquad (3)$$

where, $L = diag\{l_1, l_2, \ldots, l_K\}$.

D. Beamforming design

The type of beamforming which is employed in the proposed scheme is assumed to be fixed. This type of beamforming is obtained by using the singular value decomposition (SVD), which is shown below,

$$[\mathbf{U}, \mathbf{F}, \mathbf{Q}] = svd(\mathbf{C}), \qquad (4)$$

where, U and Q represent the unitary matrices, while F represents a diagonal matrix that contains the eigenvalues of the channel matrix C.

E. Beamforming ZF receiver design

By having the matrix U, which is obtained from the SVD analysis of the channel matrix, the beamforming ZF receiver is designed as follows,

$$\mathbf{e}_k = \frac{\mathbf{u}_k}{\|\mathbf{u}_k\|}, \qquad (5)$$

where, $\|.\|$ represents the norm function. Also $\boldsymbol{u}_k$ is k-th column of U matrix.

F. Calculation of telecommunication channel matrix

According to important parameters such as path-loss, beamforming and its receiver type, the equivalent telecommunication channel matrix between users and the BS can be formulated as follows,

$$\mathbf{h}_k = \mathbf{e}_k \mathbf{c}_k^H \mathbf{t}, \qquad (6)$$

where, t represents the transmit beamforming, which is considered to be constant vector. Also c_k is k-th column of C matrix.

The type of the precoding used in the proposed method is the ZF, which is formulated as follows,

$$A = H^H \left(HH^H \right)^{-1}. \tag{7}$$

where, $H = [h_1, h_2, \ldots, h_K]$.

G. Received Signal

We define the channel matrix and the precoding scheme. Thus, the received signal of the k-th user is written as follows,

$$y_k = \sqrt{\alpha p_k}\, c_k^H a_k s_k + \sqrt{\alpha} \sum_{i=1, i \neq k}^{K} \sqrt{p_i}\, c_i^H a_i s_i + n_k, \tag{8}$$

where, α represents the normalize factor and is equal to $\alpha = \frac{1}{E\{tr(A^H A)\}}$. The parameter p_k represents the power of each user. Also, c_k is k-th column of C matrix. The symbol which sends information is shown by s_k and it has the property $E\{|s_k|^2\} = 1$. Furthermore, a_k shows the k-th columns of A matrix. Note that, n_k represents the additive white Gaussian noise with zero mean and unit variance value, i.e., $n_k \sim CN(0,1)$ [8].

H. Signal-to-Interference plus-Noise Ratio

Based on the signal which is received by users, the signal-to-interference plus-noise ratio (SINR) equals,

$$\gamma_k = \frac{\alpha p_k \left| c_k^H a_k \right|^2}{\alpha \sum_{i=1, i \neq k}^{K} p_i \left| c_i^H a_i \right|^2 + 1}. \tag{9}$$

I. Sum Data Rate

The relation of the total data rate of users with respect to SINRs is as follows,

$$r_{sum} = \sum_{k=1}^{K} r_k = \sum_{k=1}^{K} \log_2 \left(1 + \gamma_k \right). \tag{10}$$

J. Energy Efficiency

The main purpose of this paper is to maximize the EE. For this reason, a precise definition of the EE should be provided. The EE is the ratio of the sum data rate of the system to the total power consumption of users plus constant power consumption of the circuit. Hence, the EE relationship is formulated as follows,

$$\eta_k = \frac{\sum_{k=1}^{K} r_k}{\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m}} = \frac{\sum_{k=1}^{K} \log_2 \left(1 + \gamma_k \right)}{\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m}}, \tag{11}$$

where, P_c represents the constant power of the circuit. Constant power of circuit includes power dissipation in the baseband processing, the transmission filter, and the digital to analog converter.

III. PROPOSED SOLUTION AND ITERATIVE ALGORITHM

One of the most important challenges in the 5G telecommunication networks is the issue of reaching high EE values. We deals with this issue in detail. This section is divided into two general subsections. The first part is dedicated to the formulation of optimization problem and introduces its constraints. In the second part, an efficient solution to this optimization problem is presented, and finally two iterative algorithms to numerically solve this optimization problem are introduced.

A. Problem Formulation

In this subsection, the optimization problem is introduced. The purpose of this optimization is to maximize the EE value of the system. This maximization must be accomplished under two practical constraints. These two constraints include the maximum transmission power, which means that the total power consumed by users should not be more than the maximum transmission power. The second constraint is the minimum user data rate, which means that each user's data rate must be higher than the minimum data rate considered by the system. We have,

$$\max_{\{p_1, p_2, \ldots, p_K\}} \eta_{EE} = \max_{\{p_1, p_2, \ldots, p_K\}} \frac{\sum_{k=1}^{K} r_k}{\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m}} \tag{12.a}$$

$$C1: \sum_{k:d_k < \frac{D}{2}}^{K} p_k \leq \xi P_{max} \tag{12.b}$$

$$C2: \sum_{k:d_k > \frac{D}{2}}^{K} p_k \leq (1-\xi) P_{max} \tag{12.c}$$

$$C3: r_k \geq R_{k,min}, \tag{12.d}$$

where, $C1$ and $C2$ represent the maximum transmission power for users of the first group (users with strong channel conditions) and users of the second group (users with weak channel conditions), respectively. $C3$ also indicates the minimum user data rate. $\xi = \frac{\Sigma_{k:d_k < D/2}^{K} d_k}{\Sigma_{k=1}^{K} d_k}$ is the allocation coefficient of total transmission power to the users of the first group. In (12), $R_{k,min}$ is the minimum user data rate and P_{max} denotes the maximum transmission power.

B. Proposed Solution

Here, a solution method for the optimization problem is provided and two iterative algorithms are presented as for its numerical solution. The reason for choosing two algorithms for numerical results is to reach a better understanding of the system performance. The optimization problem is a fractional and non-convex problem and also has two constraints that make it difficult to solve. The problem should first be converted to a convex problem, and then its constraints should be eliminated to solve it.

1) Turning the optimization problem into a convex problem

The problem (12.a) can be rewritten by using the nonlinear fraction programming and its features as well as lower bound user data rate. The EE maximization is performed if and only if we have [9]:

$$\max_{\{p_1,p_2,...,p_K\}} \left\{ \sum_{k=1}^{K} r_k - q^* \left(\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m} \right) \right\} = 0, \quad (13)$$

where, q^* represents the maximum EE.

To narrow the optimization problem, a low user data rate is required. Using the features of full channel mode information and rail feed, the lower bound of user data rate is obtained using (10) as follows [10]-[11],

$$\tilde{r}_k = \log_2 \left(\frac{\alpha p_k |\mathbf{c}_k^H \mathbf{a}_k|^2}{\alpha \sum_{i=1,i\neq k}^{K} p_i |\mathbf{c}_k^H \mathbf{a}_i|^2 + 1} \right). \quad (14)$$

According to equations (13) and (14), the optimization problem is rewritten in a convex form as follows,

$$\max_{\{p_1,p_2,...,p_K\}} \left\{ \sum_{k=1}^{K} \tilde{r}_k - q \left(\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m} \right) \right\} \quad (15.a)$$

$$C1: \sum_{k:d_k<\frac{D}{2}}^{K} p_k \leq \zeta P_{\max} \quad (15.b)$$

$$C2: \sum_{k:d_k>\frac{D}{2}}^{K} p_k \leq (1-\xi) P_{\max} \quad (15.c)$$

$$C3: r_k \geq R_{k,\min}. \quad (15.d)$$

where, q is the EE value.

To prove the convexity of (15), it can be argued that $q\left(\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m}\right)$ is an affine function and $\sum_{k=1}^{K} \tilde{r}_k$ is also a concave function; thus, it is proved that (15) is a convex optimization problem [11].

2) Elimination of constraints

In the previous part, the optimization problem was rewritten in a convex problem form. However, it is not possible to solve the problem easily due to its constraints. To resolve this issue, the Lagrange dual function is beneficial. The Lagrange dual function converts the problem into an unconstraint one, or eliminates its constraints. To do this, we consider ϕ as the Lagrange dual function of the problem as follows [12],

$$\phi(\mathrm{P},\omega_1,\omega_2,\boldsymbol{\rho}) = -\left[\sum_{k=1}^{K} \tilde{r}_k - q \left(\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m} \right) \right]$$
$$- \omega_1 \left(\zeta P_{\max} - \sum_{k:d_k<\frac{D}{2}} p_k \right) \quad (16)$$
$$- \omega_2 \left((1-\zeta)P_{\max} - \sum_{k:d_k>\frac{D}{2}} p_k \right) - \sum_{k=1}^{K} \rho_k \left(\tilde{r}_k - R_{k,\min} \right).$$

With a little mathematical simplification, the following equation is obtained,

$$\phi(\mathbf{P},\omega_1,\omega_2,\boldsymbol{\rho}) = \quad (17)$$

$$-\left[\sum_{k=1}^{K} (1+\rho_k) \tilde{r}_k - q \left(\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m} \right) \right]$$
$$-\omega_1 \left(\zeta P_{\max} - \sum_{k:d_k<\frac{D}{2}} p_k \right)$$
$$-\omega_2 \left((1-\zeta)P_{\max} - \sum_{k:d_k>\frac{D}{2}} p_k \right) - \sum_{k=1}^{K} \rho_k R_{k,\min},$$

where, ω_1 and ω_2 are the Lagrange coefficients related to the maximum transmission power of the first and second group of users, respectively. Furthermore, ρ_k is the Lagrange coefficient related to the minimum rate of users and $\boldsymbol{P}$ shows acceptable power set for users.

3) Calculate power of users

Given the relation (17) which relates to a convex and unconstraint problem, the solution is easy. To find the optimal power of users, we calculate derivative of the function ϕ and then set it to zero [13], i.e.,

$$\frac{\partial \phi}{\partial p_k} = 0. \quad (17)$$

The following equation is obtained for the optimal power of users in the first group,

$$\frac{1+\rho_k}{p_k \ln 2} - \sum_{i=1,i\neq k}^{K} \frac{1+\rho_i}{\left(\alpha \sum_{j=1,j\neq i}^{K} p_j |\mathbf{c}_j^H \mathbf{a}_j|^2 + 1 \right) \ln 2} - \omega_1 - q = 0. \quad (18)$$

According to (18), the optimum power for the first group of users is obtained. We have,

$$p_k = \frac{1+\rho_k}{\left(\sum_{i=1,i\neq k}^{K} \frac{1+\rho_i}{\left(\alpha \sum_{j=1,j\neq i}^{K} p_j |\mathbf{c}_j^H \mathbf{a}_j|^2 + 1 \right) \ln 2} + \omega_1 + q \right) \ln 2}. \quad (19)$$

Similarly, the following equation is obtained for the optimal power of the users in the second group,

$$\frac{1+\rho_k}{p_k \ln 2} - \sum_{i=1,i \neq k}^{K} \frac{1+\rho_i}{\left(\alpha \sum_{j=1,j \neq i}^{K} p_j \left| c_j^H a_j \right|^2 +1 \right) \ln 2} - \omega_2 - q = 0. \quad (20)$$

Based on (20), the optimum power for the first group of users is calculated and we have,

$$p_k = \frac{1+\rho_k}{\left(\sum_{i=1,i \neq k}^{K} \frac{1+\rho_i}{\left(\alpha \sum_{j=1,j \neq i}^{K} p_j \left| c_j^H a_j \right|^2 +1 \right) \ln 2} + \omega_2 + q \right) \ln 2}. \quad (21)$$

Equations (19) and (21) are closed form relations for calculating the optimal power for two groups of users. In this way, the optimal power of users can be easily calculated. Given these relationships, we propose iterative algorithms to solve the optimization problem in order to prove our claim that the proposed method works well.

4) Iterative Algorithms

To solve the optimization problem numerically, two algorithms are introduced to show the system performance.

Algorithm 1 is presented in the following, where n is the number of iterations of the algorithm. Furthermore, θ and ψ are positive steps for finding the optimal value of Lagrange coefficients. The parameter τ also indicates the convergence threshold of the optimization problem. The proof of optimality of this power allocation scheme for users by this algorithm is given in the appendix.

Algorithm 1: Find the optimal power of users to maximize the EE.

Initialization for parameters such as Lagrange coefficients (ω, ρ), minimum user data rate $(R_{k,min})$ and acceptable power for each user (P).

Calculate the initial value of EE $q^{(0)} = \frac{\sum_{k=1}^{K} r_k}{\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m}}$

While $\sum_{k=1}^{K} \tilde{r}_k^{(n)} - q^{(n)}(\sum_{k=1}^{K} p_k + \sum_{m=1}^{M} P_{c,m}) > \tau$ **do**

   for i=1:K **do**
      If $(d_k < D/2)$
        solve p_k according to formula (19),
      Else $(d_k > D/2)$
        solve p_k according to formula (21),
   End for
Update
$p_k^{(n)} = p_k^{(n+1)}$

$$\omega_1^{(n+1)} = max\left(0, \omega_1^{(n)} - \theta_1\left(\xi P_{max} - \sum_{k:d_k<\frac{D}{2}} p_k \right) \right)$$

$$\omega_2^{(n+1)} = max\left(0, \omega_2^{(n)} - \theta_1\left((1-\xi)P_{max} - \sum_{k:d_k>\frac{D}{2}} p_k \right) \right)$$

$$\rho_k^{(n+1)} = max\left(0, \rho_k^{(n)} - \psi(\tilde{r}_k^{(n)} - R_{T.k}) \right)$$

$$q^{(n+1)} = \frac{\sum_{k=1}^{K} \tilde{r}_k^{(n)}}{\sum_{k=1}^{K} p_k^{(n)} + \sum_{m=1}^{M} P_{c.m}}$$

$n = n+1.$
End while
 End

Algorithm 1 works in such a way that first the initial values are defined and then according to these initial values, the data rate and the EE value are calculated to enter the loop of finding the optimal power of users. Depending on how the users are placed in the cell, their optimal power is obtained by (19) and (21).

At the next step, the power of each user, the Lagrange coefficients related to the users of the first and second groups, the Lagrange coefficient related to the data rate, and the EE value are updated. According to these updates, if the convergence condition of the optimization problem is satisfied, then the power obtained in the last step is the optimal power of the users, otherwise this loop is repeated again until that the convergence condition is met.

In the following, we show Algorithm 2. The mechanism of working this algorithm is similar to the Algorithm 1, except that the EE update and the initial values which are different from Algorithm 1. Algorithm 2 is presented in the following.

Algorithm 2: Find the optimal power of users to maximize the EE.

Initialize 2: $= 0$, $v = \frac{1}{\ln 2}\underbrace{\min}_{k} \{\|c_k\|^2\}$, $\varepsilon > 0$ $p_k = 0$ $(k = 1.2 \dots K)$ transmit power and Lagrangian multipliers P_{max} , $\rho^{(0)}, \omega_1^{(0)}, \omega_2^{(0)}$.

Step 1:
for i=1:K **do**
   If $(d_k < D/2)$
      solve p_k according to formula (19),
   Else $(d_k < D/2)$
      solve p_k according to formula (21),
End if
Step 2: Let $\eta_{EE} = \frac{u+v}{2}$,
 Then a realistic EE **q** can be got by formula (11),
   If $(\eta_{EE} \geq \eta_{EE}^R)$,
      then $u = \eta_{EE}$
   Else $v = \eta_{EE}$.
   End if
Step 3: Update
$p_k^{(n)} = p_k^{(n+1)}$

$$\omega_1^{(n+1)} = max\left(0, \omega_1^{(n)} - \theta_1\left(\xi P_{max} - \sum_{k:d_k<\frac{D}{2}} p_k \right) \right)$$

$$\omega_2^{(n+1)} = max\left(0, \omega_2^{(n)} - \theta_1\left((1-\xi)P_{max} - \sum_{k:d_k>\frac{D}{2}} p_k \right) \right)$$

$$\rho_k^{(n+1)} = max\left(0, \rho_k^{(n)} - \psi(\tilde{r}_k^{(n)} - R_{T.k}) \right)$$

Step 4:
If $(v - u < \varepsilon)$
   $q \approx \frac{u+v}{2}$

Else go to Step 2.
End if

$n= n+1$

End

Algorithm 2 works in such a way that first the range in which it is possible to have the value of EE is determined and the power of users is considered to be zero. A value within the specified interval is considered as the initial value of EE. Then, we calculate the optimal values of users' power based on how they are placed in the cell and specify a new range for the EE. In the next step, all of the parameters that were initially set will be updated. In each iteration of this loop, the specified interval changes to achieve the desired value of EE.

The expectation from the numerical results of implementation of these two algorithms is that the achieved values of the EE that reach a convergence have very close behavior to each other, but the path they reach to converge may be different. Both algorithms have approximately a same performance and are expected to have similar numerical results.

IV. SIMULATION RESULTS

In this section, the performance of the system is evaluated. Numerical results presented in this section are output results obtained from two algorithms which are described. By using the numerical results of two algorithms we provide a better understanding of the system performance. Based on reported results, the values of system performance improvement in each parameter are clearly determined. There are two scenarios for each of the algorithms. The first scenario involves the proposed method and the second scenario includes the proposed method without using the cell division technique. Another scenario to compare with these results is known as the equal power allocation scheme. The proposed system is simulated in the MATLAB software environment. Also, the results shown in this section are average of the output from 500 times repetitions of the algorithm. Hence, momentary changes or randomness of the channel matrix do not cause significant variations in the system performance, which provide an accurate understanding of the system performance.

The parameters utilized in the simulation process and in the description of the proposed algorithms that have been initialized are given in Table I. The logic of these initial values is inspired by [14]-[19]. The placement of users in the cell each time the algorithms are repeated and it is done randomly. Since the radius of the cell is considered to be 500 meters, the distance between users and the BS can be in the range of (0,500). In all plotted figures, the initial values are considered to be the same value and if they are changed, it is clearly expressed.

Figure 1 shows how the EE of the system works by increasing the minimum user data rate. As it can be seen, increasing the minimum user rate reduces the EE value. As it is expected, the use of cell division technique improves system performance.

The numerical results of two proposed algorithms are very close to each other. A very small difference is seen in the output results of two algorithms that is due to the algorithm convergence mechanism. The strong point that can be mentioned is that the results of both algorithms in two modes of cell division and without cell division are better than the equal power allocation scheme. Another strength is that at the point $R_{k,min} = 6$ bit/s/Hz, when the EE reaches to zero value, the results of Algorithm 1 under the cell division mode are close to zero and have a value.

Table I. Important Parameters.

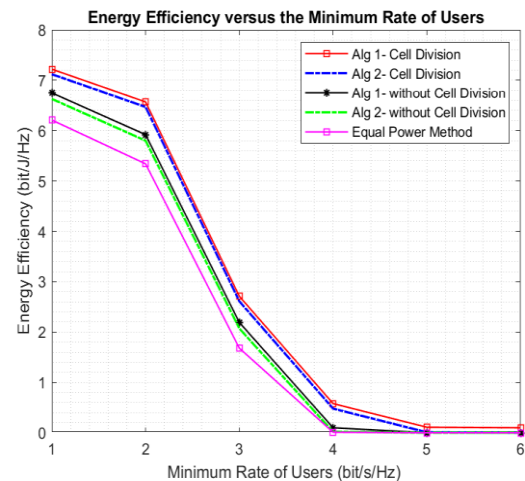| Parameters	Value
Radius of the Cell	500 m
Number of BS Antennas	3
Number of Users	3
Number of Users Antennas	1
Minimum rate of Users	2 bit/s/Hz
Reference Distance	10 m

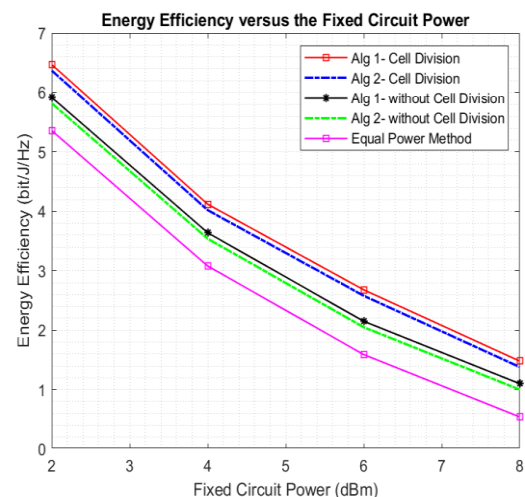Figure 2. The EE performance versus $R_{k,min}$.

Figure 3. The EE performance versus P_c.

Figure 2 also deals with the system performance in terms of the system EE. Numerical results show that by increasing the constant power of the circuit, the EE value of the system also decreases. In this figure, the numerical results of two algorithms are not significantly different from each other. The results of both algorithms in both cases, with and without the cell division technique, are better than the equal power allocation method which confirms efficient performance of both algorithms.
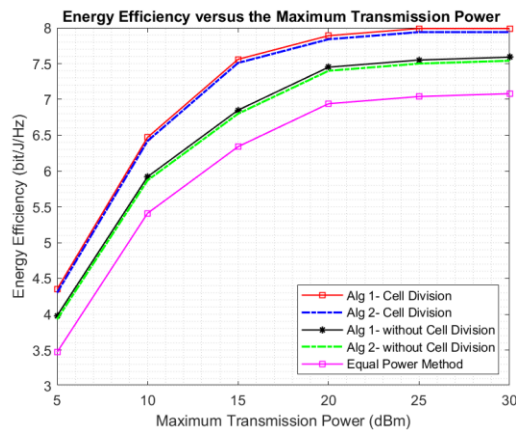

Figure 4.   The EE performance versus P_{max}.

Figure 3 depicts the EE variations of the system when the maximum transmission power of the system increases. One of the important points is that in this figure, by increasing the maximum transmission power, the EE value of the system converges, and from one point onwards, increasing the maximum transmission power no longer improves the EE of the system. The numerical difference between the two algorithms in this figure is not much significant, and it is proved that both algorithms have the same performance. Furthermore, the results of both algorithms in both scenarios are better than the equal power allocation strategy.

V.   CONCLUSION

In this paper, the performance of a MIMO-NOMA system is investigated. The optimization problem which is related to maximizing the EE value. The encountered optimization problem is difficult to solve. By using nonlinear fractional programming properties and the Lagrange dual function it becomes a tractable and solvable problem. To improve system performance, users are divided into two groups. The first group includes users whose channel conditions are strong, or in other words, their distance to the BS is less than other users. The second group is related to users who have poor channel conditions, or in other words, their location is near the edge of the cell. To better understand the system performance, two algorithms as the numerical solutions are proposed. Numerical and simulation results that are obtained from implementations confirms our claim that the cell division method improves the system performance. It is shown that there is no significant difference in the

results obtained from two proposed algorithms and the performance of both algorithms is efficient. The strength of these two algorithms is also shown due to the fact that in both cases of cell division and without cell division perform better than the method of equal power allocation scheme. What researchers can do next to this research is to reduce the number of iterations of the proposed algorithms and the time which is spent on the allocating power to users and design faster practical algorithms.

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APPENDIX

If we call the right sides (19) and (21) as $T_i(p_k)$, several properties is proved for this function, which is presented in what follows.

1) **Positive feature:** We write the relationship as follows:

$$T_1(p_k) = \cfrac{1+\rho_k}{\left(\sum_{i=1,i\neq k}^{K} \cfrac{1+\rho_i}{\left(\alpha \sum_{j=1,j\neq i}^{K} p_j \left| \boldsymbol{c}_j^H \boldsymbol{a}_j \right|^2 + 1 \right)\ln 2} + \omega_1 + q \right)\ln 2}, \quad (22)$$

$$T_2(p_k) = \cfrac{1+\rho_k}{\left(\sum_{i=1,i\neq k}^{K} \cfrac{1+\rho_i}{\left(\alpha \sum_{j=1,j\neq i}^{K} p_j \left| \boldsymbol{c}_j^H \boldsymbol{a}_j \right|^2 + 1 \right)\ln 2} + \omega_2 + q \right)\ln 2}. \quad (23)$$

Given the parameters that appear in these two equations, this relationship is always positive because all the parameters used are positive.

2) **Strictly descending property:** If we obtain the derivatives from the desired functions, then we have,

$$\frac{\partial T_1(p_k)}{\partial p_k} = \cfrac{-(1+\rho_k)}{\ln 2 \left(\sum_{i=1,i\neq k}^{K} \cfrac{1+\rho_i}{\left(\alpha \sum_{j=1,j\neq i}^{K} p_j \left| \boldsymbol{c}_j^H \boldsymbol{a}_j \right|^2 + 1 \right)\ln 2} + \omega_1 + q \right)^2}, \quad (24)$$

$$\frac{\partial T_2(p_k)}{\partial p_k} = \cfrac{-(1+\rho_k)}{\ln 2 \left(\sum_{i=1,i\neq k}^{K} \cfrac{1+\rho_i}{\left(\alpha \sum_{j=1,j\neq i}^{K} p_j \left| \boldsymbol{c}_j^H \boldsymbol{a}_j \right|^2 + 1 \right)\ln 2} + \omega_2 + q \right)^2}. \quad (25)$$

Because the numerator of the fraction is negative and the denominator of the fraction is positive, it has a power of 2; thus, it has a negative sign. This function will be always negative and hence, it is strictly descending.

3) **Scalability:** Since the desired function is a uniform descending function, by considering a random number $\varsigma > 1$, it is proved:

$$T(\varsigma p_k) < T(p_k) < \varsigma T(p_k). \quad (26)$$

Given the above relation, the scalability property is proved [15].

Due to the fact that the desired function is always a descending and positive function, with the designed algorithm and updating the Lagrange coefficients, the minimum amount of power of each user is reached. Then, minimal user power maximizes the EE value. This assumption is correct if the quality of service (QoS) is met in the system.

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