A Novel Strategy-Proof Auction Mechanism for Hybrid Spectrum Allocation

Mojtaba Mazoochi

Department of Electrical Engineering, Science & Research Branch, Islamic Azad University
Tehran, Iran
m.mazoochi@srbiau.ac.ir

Mohammad Ali Pourmina

Department of Electrical Engineering, Science & Research Branch, Islamic Azad University
Tehran, Iran
pourmina@srbiau.ac.ir

Hamidreza Bakhshi

Department of Electrical Engineering, Shahed University
Tehran, Iran
bakhshi@shahed.ac.ir

Hamidreza Navidi

Department of Applied Mathematics and Computer Science, Shahed University
Tehran, Iran
navidi@shahed.ac.ir

Abstract— **Auctions have been widely studied as an efficient approach of allocating spectrum among secondary users in recent years. On the other side, a wide range of frequency bands could be available in a spectrum auction considering the current trend of deregulating wireless resources, therefore, channels provided by the primary users may reside in widely separated frequency bands, and due to the difference in propagation profile, would show significant heterogeneity in transmission range, channel error rate, path-loss, etc. Also, we can consider the channels with similar propagation and quality characteristics, for example, channels located in the same frequency band, are homogeneous and can be located in one spectrum type. Therefore, in this paper, we propose a novel double auction mechanism for both homogeneous and heterogeneous spectrums, called hybrid spectrums. The hybrid auction design has its own challenges, especially it also inherits the challenges related to heterogeneity. We prove that our auction design can not only solve the challenges caused by hybrid spectrums but also preserve three important economic aspects including truthfulness, budget balance and individual rationality.**

Keywords- Spectrum Allocation; Double Auction; Heterogeneity; Strategy-Proofness; Hybrid Spectrum

I. INTRODUCTION

Spectrum is a valuable, scarce and finite natural resource that is needed for many different applications. Rapid growth in cellular mobile and wireless broadband alongside developments in other areas like broadcasting, utilities, and innovation in machine to machine applications have resulted in increased demand for spectrum for a variety of uses. Ever-increasing wireless traffic demand has contributed to the spectrum crisis. On the other hand, the traditional exclusive licensing spectrum policy emphasizes on effectively protecting the wireless users from interfering with each other. However, spectrum occupancy measurements in various countries have indicated that a significant amount of the licensed spectrum remains unused in many places much of the time [1-3].

Auctions have been widely studied as an efficient approach of allocating spectrum among secondary

users in recent years. However, most existing works consider spectrums as identical items. In other words, they assume that the channels are homogenous to buyers so that their requests are not channel specific. The channels provided by the primary users may reside in widely separated frequency bands, and due to the difference in propagation profile, would show significant heterogeneity in transmission range, channel error rate, path-loss, etc. We consider a double auction scheme with both homogeneous and heterogeneous spectrums, called hybrid spectrums. In the auction, secondary users would be able to evaluate the value/utility of different spectrum types and specify a particular spectrum type they need in their requests. For example, some wireless users only request the spectrums residing in lower-frequency bands due to the limitation of wireless devices.

In the other hand, truthfulness is a critical factor to attract participation. An auction could be vulnerable to market manipulation and produce very poor outcomes if this property is not guaranteed [4]. In this paper, we propose a novel truthful double auction scheme for hybrid spectrum to cope with the aforementioned challenges. To tackle reusability in hybrid spectrum transactions, since the conflict relationships between one pair of buyers are non-identical in different frequencies, we employ a grouping procedure considering non-identical interference graphs to form non-conflict buyer groups. To the best of our knowledge, our proposed auction mechanism is the first double auction mechanism for hybrid spectrum transactions.

The rest of the paper is organized as follows. We provide the related work in section II. Section III formulates the problem of hybrid spectrum exchange between spectrum owners and spectrum demanders as a double auction. In section IV, challenges in hybrid auction design are more explained. We give the description of our auction mechanism for hybrid spectrums in section V. We prove that our auction mechanism has three economic properties in section VI. Finally, we draw conclusions and point out possible future work in section VII.

II. RELATED WORK

Auctions have been widely studied as an efficient approach of allocating spectrum among secondary users in recent years. Many works follow on the designs of spectrum auctions in different scenarios [5-22].

Truthfulness is a critical factor to attract participation. Although the first efforts in the scope of spectrum auctions had not considered truthfulness, such as [5-7], authors in [8] proposed the first truthful spectrum auction design, but only addressed single-sided auctions. Then, the first truthful double auction design with spectrum reuse, called TRUST, is proposed in [9]. Later, authors in [10-13], proposed some strategy-proof single-sided multi-channel auction mechanisms. Also, some truthful double auction mechanisms proposed in [14-16]. Different from periodic auction model, some works study the spectrum allocation in an online model [17-19]. In an online spectrum auction, buyers may arrive at different

times and they can request the spectrum for a particular duration. However, most existing works consider spectrums as identical items. Some different aspects of heterogeneity in the scope of spectrum auctions are studied in [20-22]. Authors in [21], proposed an auction design in TV white spaces areas with non-identical objects taking the bandwidth and power requirements of the secondary users into account. In this paper, the spectrum allocation problem has been defined as an optimization problem where maximum payoff of the spectrum broker is the optimization target. However, [21] is not a double auction scheme and its design target is different from our design targets. Also, a double auction design for cooperative communications with heterogeneous relay selections is proposed in [22]. However, reusability is not considered in this paper. Unfortunately, the most of existing designs either do not consider the various aspects of heterogeneity or assume only the scenario where each seller contributes one distinct channel and each buyer would like to purchase one channel [20].

III. PRELIMINARIES AND PROBLEM DESCRIPTION

A. Problem formulation

We formulate the problem of hybrid spectrum exchange between spectrum owners and spectrum demanders as a double auction. We consider the scenario where N secondary service providers (called buyers) trying to purchase spectrum resources with K various types from M spectrum owners (called sellers). We consider a single-round double spectrum auction with one auctioneer, M sellers, and N buyers. Let $S = \{s_1, s_2, ..., s_M\}$ denotes the set of sellers, $W = \{w_1, w_2, ..., w_N\}$ denotes the set of buyers and $T = \{t_1, t_2, ..., t_K\}$ denotes the set of spectrum types. We assume that each seller can contribute multiple channels with various spectrum types and each buyer can obtain multiple channels with various spectrum types. Also, the channels with the same spectrum type can potentially be reused by multiple non-conflicting buyers to achieve high spectrum efficiency. We also assume that the channels with the same spectrum type are homogeneous, but the channels with the different spectrum types are heterogeneous, so each buyer has different valuations for the channels with various spectrum types, but its valuations are the same for the channels with the same spectrum type. We allow buyers to express their preferences over each spectrum type separately. Therefore, the buyers' bids are spectrum type-specific. We also assume that the auction is sealed-bid, private and collusion-free.

Each buyer submits a vector of bids; one for each spectrum type. We denote $B_n^b = \{b_{n1}^b, b_{n2}^b, ..., b_{nK}^b\}$ as the bid vector of buyer n for the available spectrum types and B^b as the bid matrix of all buyers. Also $B_m^s = \{b_{m1}^s, b_{m2}^s, ..., b_{mK}^s\}$ is the bid vector of seller m for the available spectrum types and B^s as the bid matrix of all sellers. We represent the true valuation of seller s_m and buyer w_n for the spectrum type t_k by v_{mk}^s and v_{nk}^b, respectively.

In the auction, the auctioneer determines the payment P_{mk}^s for seller s_m if it wins a channel with type t_k and the price P_{nk}^b that buyer w_n should pay if it wins a channel with this type. Therefore, the utility of

seller s_m for spectrum type t_k is $U_{mk}^s = P_{mk}^s - v_{mk}^s$ if it wins the auction for a channel with type t_k, and 0 otherwise. Also the utility of buyer w_n for spectrum type t_k is $U_{nk}^b = P_{nk}^b - v_{nk}^b$ if it wins the auction for a channel with type t_k, and 0 otherwise.

B. Economic properties

Truthfulness, individual rationality and budget balance are critical properties required to design economically robust double auctions. We now formally describe these properties [23]:

Definition 1 (Truthfulness): A double auction is truthful (or strategy-proof) if and only if no seller *m* or buyer *n* can improve its own utility by bidding untruthfully ($B_{mk}^s \neq v_{mk}^s$ or $B_{nk}^b \neq v_{nk}^b$):

$$U_{mk}^s(v_{mk}^s) \geq U_{mk}^s(B_{mk}^s),\ U_{nk}^b(v_{nk}^b) \geq U_{nk}^b(B_{nk}^b), \forall s_m \in S,$$
$$\forall w_n \in W, \forall t_k \in T \qquad (1)$$

Definition 2 (Individual Rationality): A double auction is individual rational if no winning seller is paid less than its bid and no winning buyer pays more than its bid:

$$P_{mk}^s \geq B_{mk}^s, P_{nk}^b \leq B_{nk}^b, \forall s_m \in S, \forall w_n \in W, \forall t_k \in T \quad (2)$$

Definition 3 (Ex-Post Budget Balance): A double auction is ex-post budget balanced if the auctioneer's profit is non-negative. The profit is defined as the difference between the revenue collected from buyers and the expense paid to sellers:

$$\Phi = \sum_{\substack{1 \leq n \leq N \\ 1 \leq k \leq K}} P_{nk}^b - \sum_{\substack{1 \leq m \leq M \\ 1 \leq k \leq K}} P_{mk}^s \geq 0 \qquad (3)$$

This property ensures that the auctioneer has incentive to setup the auction.

IV. CHALLENGES OF HYBRID SPECTRUM AUCTION DESIGN

In this section, we first introduce different spectrum types including homogeneous, heterogeneous and hybrid, and then briefly explain the challenges of auction mechanism design for hybrid spectrum.

A. Spectrum types

Since a wide range of frequency bands could be available in a spectrum auction, channels provided by the primary users may reside in widely separated frequency bands, and due to the difference in propagation profile, would show significant heterogeneity in transmission range, channel error rate, path-loss, etc. Also, there could be the channels with similar propagation profile. We introduce the spectrum type *ST* so that each spectrum type includes the channels with similar propagation and quality characteristics, for example, the channels located in the same frequency band. Since that the spectrums provided by the primary users can be grouped into different spectrum type sets based on their propagation and quality characteristics we can partition the whole candidate spectrums into several homogeneous spectrums each with similar type. Of course, the spectrums with different types are heterogeneous. Therefore, we consider a double auction scheme with both homogeneous and heterogeneous spectrums, called hybrid spectrums.

B. The challenges of auction mechanism design for hybrid spectrum

In this section, we briefly illustrate the challenges of designing an auction mechanism for hybrid spectrum with the desired targets. Obviously, some of these challenges in hybrid spectrum auction are related to the heterogeneity property in the frequencies located in the various spectrum types.

1) Heterogeneity in spectrum availability

Spectrum availability may vary by location. It means that some channels provided by the primary users could be not available in the locations of some buyers, for example, because of geography limitations in the license of primary users or occupying of these channels in these locations by other users and so on. Buyer grouping is applied in most traditional auction design to achieve spectrum availability; since two not interfering buyers with any common available channels should not be grouped together it is required to consider this type of heterogeneity in our grouping procedure. So we consider the heterogeneity in spectrum availability in creating interference graphs in our hybrid spectrum auction mechanism.

2) Heterogeneity in transmission range of frequencies

It means that different frequencies have different path losses and therefore, different transmission ranges such that we have:

$$L \propto 10 \log f^2 \qquad (4)$$

where L is the total path loss in decibel and f is the transmission frequency in megahertz [24]. Since in our model, we suppose that the spectrums offered by spectrum owners may consist of a wide range of frequencies, so the interference relationships among spectrum buyers in different channels are non-identical. Fig. 1 and Fig. 2 show an example of such non-identical relationships for two different frequencies.

Of course, these interference relationships are more complex in hybrid spectrum condition. We propose a novel grouping procedure considering non-identical interference graphs to group hybrid channels.

3) Heterogeneity in buyers' valuation for different spectrum types

It means that the valuations of a buyer for different frequency types are different. For example, some secondary users only request the spectrums residing in lower-frequency bands due to the limitation of wireless devices. On the other hand, we assume that the valuations of every buyer for the frequencies of belong to the same spectrum type are equal due to the similar propagation profiles. Also, spectrum buyers may express different preferences for different spectrum types. Therefore, it is required to redesign the existing winner selection and pricing algorithms appropriately with the hybrid spectrum auction.
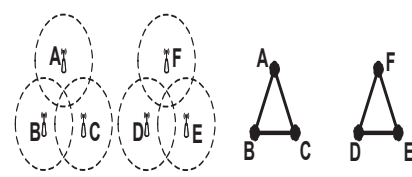

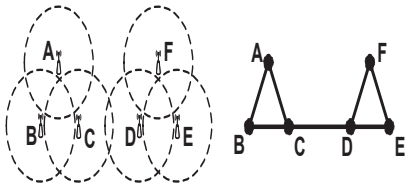Fig 1. Transmission ranges and related interference graph for freq. f_1

Fig 2. Transmission ranges and related interference graph for freq. f_2 so that $f_2 \ll f_1$

4) Achieving the economical properties for hybrid spectrum auction

To the best of our knowledge, our proposed auction mechanism is the first double auction mechanism for hybrid spectrum transactions and so existing mechanisms are unsuccessful in meeting our design targets when directly applied to hybrid spectrum auction. We design our hybrid spectrum auction mechanism so that maintain truthfulness, individual rationality and budget balance which are the critical properties required to design economically robust double auctions.

V. AUCTION PROCEDURE

Our auction mechanism consists of the following phases:

A. Buyer grouping

The buyer group formation is performed by auctioneer with a bid-independent algorithm to keep truthfulness and prevent market manipulation. The buyer's interference condition is modeled as conflict graph. On the other hand, the propagation loss on a terrestrial line-of-sight path relative to the free-space loss is the sum of different contributions. Each of these contributions has its own characteristics as a function of frequency, path length and geographic location [25-26]. We note that since channel characteristics are dependent on the frequency used, we can expect that the shape of the interference regions will be channel dependent [27], so we use channel dependent interference graph. Of course, since the channels belong to each spectrum type show similar propagation and quality characteristics, we assume that these channels are homogeneous. Also, we present and prove the following theorem for obtaining the buyer groups related to the channels belong to each spectrum type:

Theorem 1. To group the buyers belong to each spectrum type $t_i \in T$, only buyer grouping using the interference graph related to the smallest channel in this spectrum type can guarantee interference safe.□

Proof: First, we sort the channels belong to spectrum type t_i in non-decreasing order. Let h_0 be the smallest channel. We create the interference graph $G(t_i|h_0)$ related to the smallest channel h_0 and consider each two arbitrary nodes (buyers) w_j and w_k with no-edge between them. We denote $TR(w_j|h_l)$ as the transmission range of w_j using the channel h_l. According to (4), we have:

$$TR(w_j|h_l) \leq TR(w_j|h_0), TR(w_k|h_l) \leq TR(w_k|h_0), \forall h_l \in t_i \quad (5)$$

It means that if two nodes w_j and w_k do not interfere with each other in h_0, they do not interfere in

any other channel. Therefore, if we group the buyers using $G(t_i|h_0)$, interference safe will be guaranteed.

Also, we show that the buyer grouping using any other interference graph such as $G(t_i|h_l)$ so that $h_l > h_0$ can cause interference between buyers. We create the interference graphs $G(t_i|h_l)$ and consider two nodes w_j and w_k with no-edge between them, so we should have:

$$TR(w_j|h_l) + TR(w_k|h_l) < d_{ij} \quad (6)$$

where d_{jk} denotes the distance between nodes w_j and w_k.

From (5) and (6), the following condition can be established for any channel $h_s < h_l$:

$$TR(w_j|h_l) + TR(w_k|h_l) < d_{ij} < TR(w_j|h_s) + TR(w_k|h_s) \quad (7)$$

It means that two nodes w_j and w_k do interfere with each other in h_s. So our claim holds.∎

Suppose the set of channel types from M sellers is $T = \{t_1, t_2, \ldots, t_K\}$. The group formation problem for each type belongs to T is equivalent to finding the independent sets of nodes in the related conflict graph. Let $A = (a_{ij})$ be a 2-dimensional (0,1)-matrix which represents the buyers' channel type availability such that $a_{ij} = 1$ means that channel type t_j is available for buyer w_i. Also, let $C = (c_{ijk})$ be a 3-dimensional (0,1)-matrix which represents the conflict relationships between buyers for each type such that $c_{ijk} = 1$ means that buyers w_j and w_k don't interfere with each other in channel type t_i. Finally, let D^b and D^s be 2-dimensional (0,1)-matrices which represent the buyers' and the sellers' demands for each channel type belongs to T. For instance, $d_{ij}^b = 1$ means that the buyer w_i wants to buy a channel with type t_j.

In buyer grouping step, the inputs are A, C and D^b matrices with the channel types set T, which are all bid-independent. After buyer grouping, we get a family of maximal independent sets G_i^b for each channel type t_i. In other words, G_i^b contains the sets which can purchase the channel type t_i. At first, the buyer grouping obtains the set of candidate buyers Q_i for each channel type $t_i \in T$. The candidate buyers set Q_i should satisfy the following constraints:

- The channel type t_i should be available for all buyers $w_j \in Q_i$.

- All buyers in the candidate buyers set Q_i should have sent a demand for the channel type t_i.

Therefore, we have:

$$Q_i = \{w_k | w_k \in W \wedge A_{ki} = 1 \wedge D_{ki}^b = 1\} \quad (8)$$

Let h_0 be the smallest channel with type t_i belongs to the buyers in Q_i. After obtaining Q_i, the buyer grouping creates the interference graph $G(t_i|h_0)$ related to channel type t_i on the candidate buyers set Q_i according to the adjacency matrix C and then, finds the family of maximal independent sets G_i^b for the interference graph $G(t_i|h_0)$. Therefore, each set in the G_i^b is a buyer group which can purchase a channel with type t_i. We can use any existing algorithms to

find maximal independent sets, for example, the algorithms described in [28].

B. Winner selection

After the grouping phase, we can consider all buyer groups in G_i^b as the supper buyers which want to purchase the channels with type t_i. Also, we consider all sellers in G_i^s as the sellers which want to sell the channels with this type.

To determine the bid of each super buyer G_{ij}^b in G_i^b, we propose to use uniform pricing rather than discriminatory pricing in each buyer group G_{ij}^b for the same channel type t_i to make the auction individual-rational and truthful. Discriminatory pricing, such as charging each buyer proportionally to its bid, could make the auction untruthful because selective buyers in a winning group can manipulate their bids to lower their shares in the group charge while still winning the auction. Now by lowering their clearing prices, they improve their utilities, violating the truthfulness requirement. Further, this conclusion holds no matter how the group bid is computed. So we use uniform pricing in each buyer group G_{ij}^b for the same channel type t_i. We present and prove the following theorem for obtaining the buyer group bids:

Theorem 2. To make the auction individual-rational and truthful, the group bid π_{ij}^b for the channel type t_i, under per-group uniform pricing, should not be more than the product of the lowest buyer bid in the group and the number of buyers in the group.□

Proof: Let P_i^b be the price for winning buyer groups for channel type t_i. Since individual rationality requires that each buyer w_k in winning buyer group G_{ij}^b must not be charged higher than its bid for this channel type, b_{ki}^b, so under uniform pricing which the buyers in the same group are charged equally, the following inequality should be hold:

$$\frac{P_i^b}{|G_{ij}^b|} \leq b_{ki}^b \ , \forall w_k \in G_{ij}^b \qquad (9)$$

Hence we have:

$$P_i^b \leq b_{ki}^b \cdot |G_{ij}^b| \leq Min\{b_{ki}^b | w_k \in G_{ij}^b\} \cdot |G_{ij}^b| \qquad (10)$$

Also, according to the McAfee's design [29], to maintain truthfulness, the price charged to a winning group G_{ij}^b for channel type t_i is the bid of kth group G_{ik}^b for this channel type that is no higher than its own group bid π_{ij}^b:

$$P_i^b = \pi_{ik}^b \leq \pi_{ij}^b \qquad (11)$$

From (11), if we take the extreme value of P_i^b, $P_i^b = \pi_{ij}^b$, and put it in inequality (10), we have:

$$\pi_{ij}^b \leq Min\{b_{ki}^b | w_k \in G_{ij}^b\} \cdot |G_{ij}^b| \qquad (12)$$

So our claim holds.■

In winner determination phase, we first calculate the buyer group bid π_{ij}^b relating to the buyer group G_{ij}^b for channel type t_i according to theorem 2 as below:

$$\pi_{ij}^b = Min\{b_{ki}^b | w_k \in G_{ij}^b\} . |G_{ij}^b| \qquad (13)$$

Then for each channel type t_i, we sort the related buyer group bids π_i^b in non-increasing order and represent the sorted bids and its related buyer groups by π_i^{bs} and G_i^{bs}, respectively. Also we sort the seller bids π_i^s corresponding to the channel type t_i in non-decreasing order and represent the sorted bids and its related sellers by π_i^{ss} and G_i^{ss}, respectively.

We define k_i as the last profitable trade for channel type t_i:

$$k_i = argmax_{l \leq min\{length(G_i^s), length(G_i^b)\}} \pi_l^{bs} \geq \pi_l^{ss} \qquad (14)$$

Finally, the preliminary auction winners for channels with type t_i are the first $k_i - 1$ buyer groups in G_i^{bs} and the first $k_i - 1$ sellers in G_i^{ss}. Therefore, the winning buyers for channels with this type are the members of these winning buyer groups.

C. Pricing

After the winner selection phase, we calculate the price of the winning buyers and the payment of the winning sellers for each channel type t_i. To maintain truthfulness, the auctioneer pays each winning seller belongs to the winning seller set G_i^{sw} by the k_i th seller's bid π_{ki}^{ss}. Also, the auctioneer charges each winning buyer group belongs to the family of winning buyer set G_i^{bw} by the k_i th buyer group's bid π_{ki}^{bs}. Finally, this buyer's group price is shared by all the members in each winning buyer group. No charges or payments are made to losing buyers and sellers.

VI.   PROOFS OF ECONOMIC PROPERTIES

In this section, we prove that our auction mechanism has three economic properties: individual rationality, budget balance and truthfulness.

Theorem 3. The auction mechanism is individually rational.□

Proof: For each channel with type t_i and each buyer w_k in winning buyer group G_{ij}^b, according to the winner determination and pricing algorithm, we have:

$$P_{ik}^b = \frac{P_i^b}{|G_{ij}^b|} \leq \frac{\pi_{ij}^b}{|G_{ij}^b|} \leq \frac{b_{ki}^b \cdot |G_{ij}^b|}{|G_{ij}^b|} = b_{ki}^b \qquad (15)$$

Therefore, the clearing price P_{ik}^b for the winning buyer w_k and the channel type t_i is no more than its bid for this channel type, b_{ki}^b.

Also, according to the winner determination and pricing algorithm, the clearing price is the k_i th bidding price and sellers are sorted by their bidding price in non-decreasing order. Therefore, for each channel type t_i and each winning seller s_j in seller group G_i^s, we have:

$$b_{ji}^s \leq \pi_{ki}^{ss} = P_i^s = P_{ij}^s \qquad (16)$$

It means that no winning seller is paid less than its bid.

Therefore, the auction is individually rational and the theorem holds.■

Theorem 4. The auction mechanism is budget-balanced.□

Proof: According to the sorting in the winner determination and pricing algorithm, for each winning group related to channel type t_i, $\pi_i^{bs} \geq \pi_i^{ss}$ and $\left|G_i^{bw}\right| = \left|G_i^{sw}\right|$.

Therefore, $\left|G_i^{bw}\right| \cdot \pi_i^{bs} - \left|G_i^{sw}\right| \cdot \pi_i^{ss} \geq 0$.

So the auctioneer's profit Φ is always no less than zero:

$$\Phi = \sum_{1 \leq i \leq K} \left(\left|G_i^{bw}\right| \cdot \pi_i^{bs} - \left|G_i^{sw}\right| \cdot \pi_i^{ss} \right) \geq 0 \quad (17)$$

So the theorem holds. ∎

Truthfulness is a main economic property for auctions. To prove this property for our auction mechanism, we need to show that any buyer w_n and seller s_m cannot increase its utility by bidding other than its true valuation. At first we need to show that the winner determination mechanism is monotonic for buyers and sellers, and furthermore, the pricing is bid-independent. Lemma 1 to Lemma 4 prove these claims. Then we prove that our auction mechanism for hybrid spectrum is truthful or strategy-proof.

Lemma 1. Given $B^b_{-(ni)} = B^b \backslash \{b^b_{ni}\}$ and B^s, if buyer w_n wins in the auction for a channel with type t_i, it also wins by bidding $\acute{b}^b_{ni} > b^b_{ni}$ for this channel type.□

Proof: According to the winner determination algorithm, the buyer bids of relating to the other channel types other than t_i do not effect on the winner determination for the channel type t_i. Therefore, we here consider the bids of buyers other than w_n only for the channel type t_i, $B^b_{(-n)i}$. We consider two possible cases:

Case 1: If the bid of buyer w_n for the channel type t_i be greater than the related group bid of this buyer for t_i, $b^b_{ni} > \pi^b_{gi}$:

As $\acute{b}^b_{ni} > b^b_{ni}$, so the group bid will not change by bidding $\acute{b}^b_{ni}$, $\acute{\pi}^b_{gi} = \pi^b_{gi}$. Therefore, the auction result will not change and w_n will win channel type t_i in the auction.

Case 2: If $b^b_{ni} = \pi^b_{gi} = min\{b^b_{ki} | w_k \in G_{ig}\}$:

As $\acute{b}^b_{ni} > b^b_{ni}$, so the group bid will be greater by bidding $\acute{b}^b_{ni}$, $\acute{\pi}^b_{gi} > \pi^b_{gi}$. Also w_n is a winning buyer by bidding b^b_{ni}, so according to the winner determination algorithm, $\pi^b_{gi} \geq \pi^{bs}_{ki}$. Then $\acute{\pi}^b_{gi} \geq \pi^{bs}_{ki}$ and therefore, w_n will also win in the auction for the channel type t_i by bidding $\acute{b}^b_{ni}$.∎

Lemma 2. Given $B^s_{-(mi)} = B^s \backslash \{b^s_{mi}\}$ and B^b, if seller s_m wins in the auction for the channel type t_i, it also wins by bidding $\acute{b}^s_{mi} < b^s_{mi}$ for this channel type.□

Proof: According to the winner determination algorithm, the seller bids of relating to the other channel types other than t_i do not effect on the winner determination for the channel type t_i, so we consider only $B^s_{(-m)i}$.

Since s_m is a winning seller by bidding b^s_{mi}, so according to the winner determination algorithm,

$b^s_{mi} \leq \pi^{ss}_{ki}$. Then $\acute{b}^s_{mi} < \pi^{ss}_{ki}$ and therefore, s_m will also win in the auction for the channel type t_i by bidding $\acute{b}^s_{mi}$.∎

Lemma 3. Given $B^b_{-(ni)} = B^b \backslash \{b^b_{ni}\}$ and B^s, if buyer w_n wins in the auction for channel type t_i by bidding b^b_{ni} and $\acute{b}^b_{ni}$, the prices charged to w_n are the same.□

Proof: We only consider $B^b_i \backslash \{b^b_{ni}\}$ and B^s_i, because the buyer and the seller bids belong to the other channel types other than t_i do not effect on the winner determination and also pricing for the channel type t_i. Without the loss of generality, let $\acute{b}^b_{ni} > b^b_{ni}$. According to Lemma 1, increasing a winning buyer's bid will not change the auction results and also the position of k_i in the sorted list of group bids belong to channel t_i. Since the price is only dependent on position k_i, the prices charged for buyer w_k by bidding b^b_{ni} and $\acute{b}^b_{ni}$ are the same.∎

Lemma 4. Given $B^s_{-(mi)} = B^s \backslash \{b^s_{mi}\}$ and B^b, if seller s_m wins in the auction for channel type t_i by bidding b^s_{mi} and $\acute{b}^s_{mi}$, the payment paid to s_m is the same for both.□

Proof: We consider $B^s_i \backslash \{b^s_{mi}\}$ and B^b_i, because the buyer and the seller bids belong to the other channel types other than t_i do not effect on the winner determination and also pricing for the channel type t_i. Since seller m wins the auction by bidding b^s_{mi} and $\acute{b}^s_{mi}$, the payment is determined by a seller ranked after m, which does not change in both cases. Our claim holds.∎

Theorem 5. The auction mechanism is truthful for buyers.□

Proof: We need to prove that any buyer w_n cannot increase its utility for one channel with type t_i by bidding other than its valuation for this channel type, $\acute{b}^b_{ni} \neq v^b_{ni}$. Since according to the winner determination algorithm, the bids of relating to the other channel types other than t_i do not effect on the winner determination for the channel type t_i, so we consider the bids of buyer w_n only for this channel type. There are four possible cases for bidding of one buyer. In the following, we examine our claim for these cases:

Case 1: If w_n bids either truthfully or untruthfully, he loses in the auction.

In this case, since buyer w_n loses in the auction for both bids $\acute{b}^b_{ni}$ and v^b_{ni}, this buyer charged with zero for both bids, leading to the same utility of zero.

Case 2: Buyer w_n wins in the auction only if he bids truthfully.

According to Lemma 1, this case happens only if $\acute{b}^b_{ni} < v^b_{ni}$. According to Theorem 3, the clearing price for the winning buyer w_n is no more than its bid for this channel, so the utility of winning buyer w_n by bidding v^b_{ni} is non-negative. On the other hand, the utility of losing buyer w_n by bidding $\acute{b}^b_{ni}$ is zero. So our claim holds.

Case 3: Buyer w_n wins in the auction only if he bids untruthfully.

According to Lemma 1, this case happens only if $\acute{b}^b_{ni} > v^b_{ni}$. Let buyer w_k for bidding channel type t_i placed in group G^b_{ij}. Since buyer w_n wins the auction by bidding higher than v^b_{ni}, w_n should have offered the lowest bid in its group when bidding v^b_{ni}, denoted by π^b_{ij}. So we have:

$$\pi^b_{ij} = min\{b^b_{ki} | w_k \in G^b_{ij}\} . |G^b_{ij}| = v^b_{ni} . |G^b_{ij}| \qquad (18)$$

Also, since w_n wins by bidding $\acute{b}^b_{ni}$ and loses by bidding v^b_{ni}, its group bid should satisfy $\acute{\pi}^b_{ij} \geq \acute{P}^b_i \geq \pi^b_{ij}$, where $\acute{\pi}^b_{ij}$ represents the group bid of G^b_{ij} when w_n bids $\acute{b}^b_{ni}$ and $\acute{P}^b_i$ represents the price charged to this group when w_n wins in the auction by bidding $\acute{b}^b_{ni}$. so the utility of buyer w_n by bidding $\acute{b}^b_{ni}$ is:

$$v^b_{ni} - \frac{P^b_i}{|G^b_{ij}|} \leq v^b_{ni} - \frac{\pi^b_{ij}}{|G^b_{ij}|} = 0 \qquad (19)$$

Therefore, the utility of winning buyer w_k by bidding v^b_{ni} is less than or equal to zero. On the other hand, since buyer w_n loses in the auction by bidding v^b_{ni}, its utility is zero for this bid. So our claim holds.

Case 4: If w_n bids either truthfully or untruthfully, he wins in the auction.

According to Lemma 3, buyer w_n is charged by the same price for both bids $\acute{b}^b_{ni}$ and v^b_{ni}. Therefore, its utility is the same for both bids and the claim holds.

From the above cases, we result that no buyer can increase its utility by bidding untruthfully. It means our auction mechanism is truthful for buyers.∎

Theorem 6. The auction mechanism is truthful for sellers.□

Proof: Similar to the previous theorem, we need to prove that any seller s_m cannot obtain higher utility for one channel with type t_i by bidding other than its valuation for this channel, $\acute{b}^s_{mi} \neq v^s_{mi}$. Since according to the winner determination algorithm, the bids of relating to the other channel types other than t_i do not effect on the winner determination for the channel type t_i, so we consider the bids of seller s_m only for this channel type. Similarly, there are four possible cases for bidding of one seller. In the following, we examine our claim for these cases:

Case 1: If s_m bids either truthfully or untruthfully, he loses in the auction.

In this case, since seller s_m loses in the auction for both bids $\acute{b}^s_{mi}$ and v^s_{mi}, its payment is zero for both bids, leading to the same utility of zero.

Case 2: Seller s_m wins in the auction only if he bids truthfully.

According to Lemma 2, this case happens only if $\acute{b}^s_{mi} > v^s_{mi}$. According to Theorem 3, no winning seller is paid less than its bid, so the utility of winning seller s_m by bidding v^s_{mi} is non-negative. On the other hand, the utility of losing seller s_m by bidding $\acute{b}^s_{mi}$ is zero. So our claim holds.

Case 3: Seller s_m wins in the auction only if he bids untruthfully.

According to Lemma 2, this case happens only if $\acute{b}^s_{mi} < v^s_{mi}$. Let P^s_i and $\acute{P}^s_i$ represent respectively the payment to the winning sellers when s_m bids truthfully and untruthfully. Since seller s_m loses by bidding v^s_{mi}, so $P^s_i < v^s_{mi}$. Also, because the seller s_m lowers its bids and wins in the auction, so $\acute{P}^s_i \leq P^s_i$. Hence $\acute{P}^s_i \leq P^s_i < v^s_{mi}$. Therefore, utility of seller s_m when bids untruthfully is $\acute{P}^s_i - v^s_{mi} < 0$. On the other hand, the seller s_m loses in the auction by bidding truthfully, hence its utility is zero. So our claim holds.

Case 4: If s_m bids either truthfully or untruthfully, he wins in the auction.

According to Lemma 4, the payment for seller s_m does not change for both bids $\acute{b}^s_{mi}$ and v^s_{mi}. Therefore, its utility is the same for both bids and the claim holds.

From the above cases, we result that no seller can increase its utility by bidding untruthfully. It means our auction mechanism is truthful for sellers.∎

VII. CONCLUSIONS AND FUTURE WORK

In this paper, we have designed a novel truthful double auction scheme with both homogeneous and heterogeneous spectrums, called hybrid spectrums. We introduced the spectrum type concept so that each spectrum type includes the channels with similar propagation and quality characteristics. Our proposed scheme allows spectrum owners to contribute multiple channels with various spectrum types. Also, secondary service providers are able to express their preferences over each spectrum type separately. Therefore, the buyers' bids are spectrum type-specific. We illustrated the challenges of designing an auction mechanism for hybrid spectrums. We have shown that our auction design can solve the challenges caused by hybrid spectrums.

As for future work, since grouping affects on many of auction metrics such as spectrum utilization, buyer/seller satisfaction ratio and number of traded channels, we are going to offer more effective procedures for it rather than existing ones. Another possible direction is to extend the proposed mechanism to be resistant to collusion.

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Mojtaba Mazoochi received his M.Sc. degree in 1995 from Khajeh Nasir Toosi University of Technology and his B.Sc. degree in 1992 from Tehran University (Faculty of Engineering) in Electrical Engineering (Telecommunication) and is now working toward his Ph.D. degree at the Islamic Azad University with the major of system engineering. He is a Faculty Member and Director of Integrated Network Management Group in Research Institute for ICT. His research interests include wireless networks, spectrum sharing, quality of service (QoS), and network management.

Mohammad Ali Pourmina received his B.Sc. and M.Sc. degrees in electrical engineering (Telecommunications) from Islamic Azad University, Central branch, Tehran, Iran, in 1989 and 1991 respectively. He received his Ph.D. degree at Science and research Branch, Islamic Azad University, Tehran, Iran, in 1996. He joined Science and research Branch, Islamic Azad University, Tehran, Iran in 1996. He has been a member of Iran Telecommunications research center since 1992. He has performed research in the areas of packet radio networks and digital signal processing systems since 1991. His current research interests include spread-spectrum systems, cellular mobile communications, indoor wireless communications, DSP processors and wireless multimedia networks.

Hamidreza Bakhshi was born in Tehran, Iran on April 25, 1971. He received the B.Sc. degrees in electrical engineering from Tehran University, Iran in 1992, and the M.Sc. and Ph.D. degree in Electrical Engineering from Tarbiat Modarres University, Iran in 1995 and 2001, respectively. Since 2011, he is an associate professor of Electrical Engineering in Shahed University. His research interests include wireless communications, multiuser detection, and smart antennas.

Hamidreza Navidi was born in Rasht, Iran on September 17, 1965. He received his B.Sc. and M.Sc. degree in Pure Mathematics from Tehran University, Iran (respectively in 1989 and 1992) and the PhD degree in applied Mathematics from MSU in Operation Research and Game theory in 2003 (Moscow, Russian). His current research interests include Game theory and it's applications in Computer Science and IE, the optimization problems in LP and NLP. He is associate professor at the applied Mathematics and the Computer Science department of Shahed University (Tehran, Iran). He is collaborating with Department of IE of Shahed University.