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An Opportunistic Cooperative Approach for Dynamic Spectrum Leasing in Cognitive Radio Networks

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Abstract— **Spectrum leasing issue is investigated by an opportunistic cooperative approach. The scenario includes a multi-antenna primary user (working as a Base Station) and a number of single-antenna primary and cognitive radio users (CR users). The core aim of the paper is to maximize the data rate in the downlink, from the primary base station (PU BS) to the desired primary user (PU RX), taking advantage of the cooperation of opportunistically selected CR users. Cooperating CR users will also enjoy the benefits of transmitting to their intended CR receivers in a portion of time, allocated by PU BS. The main contribution of this work is proposing the opportunistic cooperative approach. Meanwhile the interference imposed on other PUs, due to cooperation of selected CR users, is removed. Moreover, as the opportunistically selected CR users form a virtual antenna array, it becomes feasible to utilize zero-forcing beamforming to remove the interference on other PUs.**

Keywords- Opportunistic Spectrum Leasing; Cognitive Radio Networks; Cooperative Communications; Stackelberg Game

I. Introduction

The rigid structure of current spectrum allocation policies creates a bottleneck for rapidly growing wireless users. On the other hand, the Federal Communication Commission (FCC) measurements reveal that most of the licensed frequency bands are either unused or utilized less than 10% of the time [1]. To address the limitations on spectrum usage, the FCC has motivated the use of opportunistic spectrum sharing to make the licensed frequency bands accessible for unlicensed wireless users. The intention behind this is to create cognitive capability of wireless devices for concurrent spectrum usage.

The concept of spectrum leasing was proposed in [2] as an approach for better spectrum utilization. In [2], based on the property-rights model of cognitive radio, the primary users (PUs) may decide to lease their own bandwidth for a fraction of time to secondary users (SUs) in exchange for cooperation in the form of distributed space-time coding. The system was modeled using Stackelberg games. In [3], the spectrum leasing idea proposed in [2] was developed and a cooperative cognitive radio framework was proposed to enable PU to lease a part of its own bandwidth to selected SUs and use them as the cooperative relay in return. A game theoretic framework for spectrum leasing was developed in [4], where PUs actively participate in a non-cooperative game with SUs. In [5], the framework provided in [4] was elaborated and a general structure for the utility functions of PUs and SUs that allows the PUs to control the price and the demand for spectrum access based on their required QoS was presented.

Transmit beamforming (TB) with receive combining is one of the simplest approaches for achieving full diversity. Compared with traditional space-time codes, beamforming and combining systems provide the same diversity order as well as significantly more array gain at the expense of requiring channel state information (CSI) at the transmitter. The issue of transmit beamfroming in cognitive radio networks (CRN) was investigated from various points of view in [6-8]. The joint problems of TB and power control in CRN were considered in [6], [7], where the objective was to optimize the sum rate of SUs, under interference constraints of PUs. The joint problems of TB at transmitter and antenna sub-set selection at receiver of a secondary network was considered in [8], where the need for the real-time CSI at the transmitter side to derive the optimal TB weight vector was removed using the Grassmannian beamforming approach presented in [9]. To provide more clarification, the authors in [9] presented a method, based on Grassmannian Line Packing technique (GLP), which does not need CSI at transmitter for TB, and works when a limited feedback is available from receiver to transmitter. The beamforming codebook in [9] is generated using GLP technique. The transmitter and the receiver preserve the same codebook, which contains, for example, M weight vectors for the Grassmannian beamforming (GB). In the GB, the index for the optimal beamforming weight vector, not the vector itself, is fed back from the receiver to the transmitter. Thus, the amount of feedback information can be reduced to $[log_2 M]$ bits.

In this paper, we add to the merit of spectrum leasing in cognitive radio networks by incorporating the distributed zero-forcing beamforming and also opportunistic cooperation of secondary users.

The main contributions of this work can be summarized according to the following:

- Implementing the idea of opportunistic cooperative spectrum leasing for cognitive radio networks in a completely new fashion.
- Proposing an optimal distributed Zero-Forcing beamforming method.

The remainder of the paper is organized as follows. The system architecture is presented in Section II. The first, second and third phases are studied in the third, fourth and fifth sections, respectively. In Section VI, the performance of the proposed solution method is evaluated using simulations and Section VII concludes the paper.

II. SYSTEM ARCHITECTURE

The system architecture consists of an primary network with $N_{PU}+1$ single-antenna primary users (PUs), including one desired PU, denoted also as PU RX, and N_{PU} PUs, and a primary base station (PU BS) equipped with M antenna and a secondary network with N_{CR} single-antenna CR user pairs, as shown in Fig. 1. Each CR user pair is comprised of a CR TX and a CR RX. In this work, the desired link is considered between PU BS and one of the single-antenna PUs, denoted as PU RX. It is further assumed that all the users use the same frequency band. Therefore, when PU BS communicates the PU RX, some interference is induced to other PUs.

We consider the same spectrum leasing model as in [2], where the spectrum leasing process is performed in three phases, as shown in Fig. 1. The PU BS is assumed to be able to grant the use of the bandwidth to a subset of opportunistically selected CR users in exchange for cooperation so as to improve the quality of the communication link to its receiver PU RX. In particular, PU BS performs transmission as shown in Fig. 1-(a). In other words, PU BS attempts to choose K optimal CR TXs out of available CR users to cooperate in serving the PU RX as relays. Meanwhile, transmit beamforming (TB) is recruited in PU BS to keep the interference on other PUs in the allowed region and also maximize the data rate in the link from PU BS to selected CR TXs. Therefore, PU BS performs the following tasks jointly: 1) selecting the qualified CR TXs to cooperate in its transmission to PU RX and, 2) finding the optimal weight vector for transmit beamforming its signals to the selected CR users. A fraction of the slot dedicated to its transmission towards the CR users is of duration $1-\alpha$ ($0 \leq \alpha \leq 1$). The remaining time α is decomposed into two subslots according to a parameter $0 \leq \beta \leq 1$. In the first subslot of duration $\alpha(1-\beta)$, the K active secondary nodes are allowed to transmit their own data and will be granted the advantage of transmission to their respective CR RXs. (Fig. 1-(b)). The last subslot is of duration $\alpha\beta$ and is used for cooperation (Fig. 1-(c)).

All channels are supposed to experience Rayleigh fading. Furthermore, channel coefficients are assumed constant during each transmission phase. $\boldsymbol{H}_{PC} \in \mathbb{C}^{N_{CR} \times M}$ denotes the channel between PU BS and all CR TXs and $\boldsymbol{H}_{CP} \in \mathbb{C}^{1 \times K}$ represents the channel between cooperating CR TXs and PU RX. In the sequel, we investigate each of the phases and determine the resource allocation in each of them.

III. FIRST PHASE: RELAY SELECTION AND TRANSMIT BEAMFORMING

A. Problem Formulation

The CR TXs who provide the highest data rate between PU BS and themselves are among the candidates. Therefore, this kind of cooperation can be categorized as an opportunistic cooperative scheme, since the multiuser diversity is utilized to improve the system performance.

In order to implement the appropriate transmit beamforming strategy, we utilize the approach presented in [8]. Therefore, we assume that the PU BS and CR TXs preserve the same codebook, which contains a number of beamforming weight vectors for the Grassmannian beamforming (GB). The beamforming codebook is generated using Grassmannian Line Packing technique (GLP) [9]. The GB does not need the CSI at PU BS to find the optimum TB weight vector and works when a limited feedback is available from CR TXs to PU BS. Therefore, the optimal TB weight vector is determined in CR TXs by selecting the best weight vector among available ones in a codebook matrix and the index of the optimal TB weight vector, not the vector itself, is fed back, using a few bits of limited feedback from the selected CR TXs to the PU BS. The received signal at CR TXs can be written as

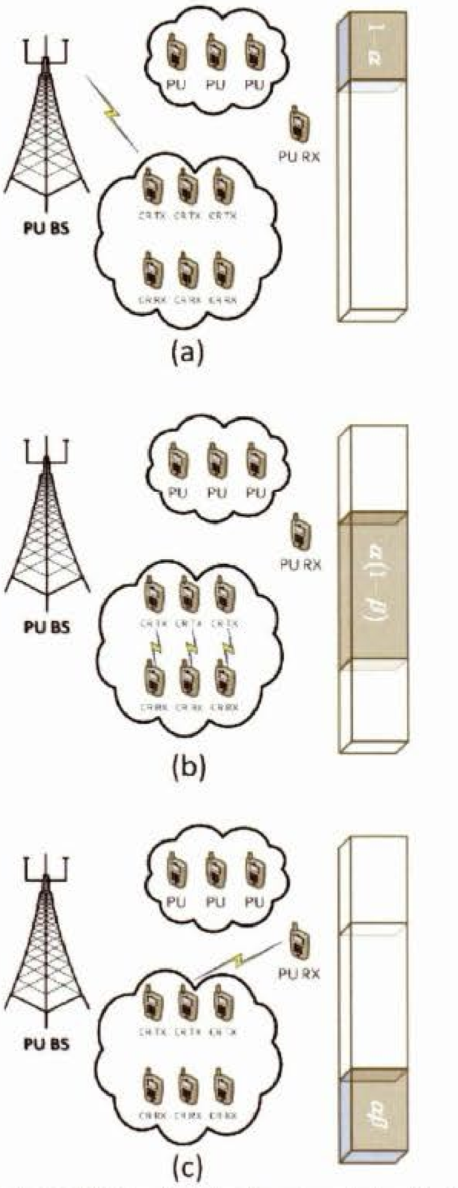

Fig. 1.System architecture including three transmission, for K = 3 secondary transmitters and receivers: (a) phase 1: primary transmission; (b) phase 2: Secondary transmission; (c) phase 3: Secondary Cooperation.

$$\boldsymbol{y} = \boldsymbol{H}_{PC}\boldsymbol{w}_{TB}x + \boldsymbol{i} + \boldsymbol{z} \qquad (1)$$

where $\boldsymbol{H}_{PC}$ denotes the channel matrix from PU BS to CR TXs; $\boldsymbol{i}$ is the interference from other PUs; $\boldsymbol{z}$ is the Zero-Mean Circularly Symmetric Complex Gaussian (ZMCSCG) noise at CR TXs; $\boldsymbol{w}_{TB}$ is the TB weight vector. The data rate from PU BS to CR TXs can be expressed as [10]

$$R_{PC} = log_2 det\left(\boldsymbol{I}_{N_{CR}} + \boldsymbol{H}_{PC}\boldsymbol{Q}_{PT}\boldsymbol{H}_{PC}^H\boldsymbol{U}^{-1}\right) \qquad (2)$$

in which $\boldsymbol{I}_{N_{CR}}$ is $N_{CR} \times N_{CR}$ identity matrix and $\boldsymbol{Q}_{PT} \in \mathbb{C}^{M\times M}$ denotes the transmit covariance matrix of PU BS. The covariance matrix of interference from other PUs imposed on CR users and noise is denoted by $\boldsymbol{U} \in \mathbb{C}^{N_{CR}\times N_{CR}}$. We assume that the total transmit power of PU BS is limited to P_{PT}. Then

$$\boldsymbol{Q}_{PT} = E\{\boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H xx^*\} = \boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H P_x \le P_{PT} \qquad (3)$$

For satisfactory operation of other PUs, interference seen at them as a result of transmission of PU BS toward PU RX should not exceed a particular threshold which is denoted by P_j $(j = 1, \cdots, N_{PU})$:

$$\boldsymbol{h}_{PP,j}\boldsymbol{Q}_{PT}\boldsymbol{h}_{PP,j}^H \le P_j,\ j = 1,2,\cdots,N_{PU} \qquad (4)$$

where $\boldsymbol{h}_{PP,j}$ denotes the channel vector form PU BS to the j-th PU. In order to select the optimal CR TXs, we define the diagonal selection matrix $\boldsymbol{S} \in \mathbb{C}^{N_{CR}\times N_{CR}}$ as [8]

$$(\boldsymbol{S})_{ii} = \begin{cases} 1 & i\text{-}th\ CR\ TX\ selected \\ 0 & otherwise \end{cases} \qquad (5)$$

CR TXs that maximize the data rates in first phase are selected as the cooperative relay. The diagonal matrix provides the index of selected CR TXs. If K CR TXs are selected ($K \le N_{CR}$), new channel matrix, $\widehat{\boldsymbol{H}}_{PC}$, with the same dimension as $\boldsymbol{H}_{PC}$, will have $N_{CR} - K$ all-zero rows. Thus, the data rate expression at PU BS can be expressed as

$$R_{PC} = log_2 det\left(\boldsymbol{I}_{N_{CR}} + \widehat{\boldsymbol{H}}_{PC}\boldsymbol{Q}_{PT}\widehat{\boldsymbol{H}}_{PC}^H\right) \qquad (6)$$

where $\widehat{\boldsymbol{H}}_{PC} = \widehat{\boldsymbol{U}}^{\frac{-1}{2}}\boldsymbol{S}\boldsymbol{H}_{PC}$, and $\widehat{\boldsymbol{U}}$ is defined as follows. With the selected cooperating CR TXs, we have reduced $K \times 1$ interference and noise vectors which gives a new interference and noise covariance matrix, $\boldsymbol{U}_{reduced}$, of dimension $K \times K$. This matrix is inflated to form $\widehat{\boldsymbol{U}}$, an $N_{CR} \times N_{CR}$ matrix, by adding rows and columns of zeros corresponding to the receive antennas not selected.

Therefore, the indices of the K optimal CR TXs for cooperating with PU BS, the optimum TB weight vector and optimum transmit power at PU BS are obtained by solving the following problem:

$$\begin{aligned} \boldsymbol{P}\!: \max_{\boldsymbol{w}_{TB},\boldsymbol{S},P_x}\ & log_2 det\left(\boldsymbol{I}_{N_{CR}} + \widehat{\boldsymbol{H}}_{PC}\boldsymbol{Q}_{PT}\widehat{\boldsymbol{H}}_{PC}^H\right) \\ s.t.\ & (\boldsymbol{S})_{ii} \in \{0,1\}, i = 1,\cdots,N_{CR} \\ & tr(\boldsymbol{Q}_{PT}) \le P_{PT} \Rightarrow tr(\boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H) \le \frac{P_{PT}}{P_x} \\ & tr(\boldsymbol{S}) = K \\ & P_x\boldsymbol{h}_{PP,j}\boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H\boldsymbol{h}_{PP,j}^H \le P_j,\ j = 1,\cdots,N_{PU} \end{aligned} \qquad (7)$$

B. An iterative method for Solving the Problem $\boldsymbol{P}$

A straightforward way to solve $\boldsymbol{P}$ is to perform an exhaustive search (ES) over all possible combinations of CR TXs and TB weight vectors and optimize over P_x. Hence, ES amounts to optimizing P_x, $\binom{N_{CR}}{K} \times \binom{M}{1}$ times subject to interference and transmit power constraints, where M denotes the number codewords in the transmit beamforming codebook matrix. Each optimization of P_x can be considered as a convex problem. However, the need to iterate through all possible combinations gives a complexity which explodes for higher dimensional systems.

The problem $\boldsymbol{P}$ is highly non-convex and can be classified as an example of an integer programming problem, since matrix $\boldsymbol{S}$ has only binary elements [11]. The non-convexity of the problem arises due to the nature of the objective function, interference and binary constraints. Further, the binary variable renders the problem NP-hard problem. In order to obtain a more computationally efficient approach, we modify the problem in the following way. The binary structure of $\boldsymbol{S}$ can be relaxed so that the user selection variable take on values in the interval 0 to 1. This makes the problem far easier to solve than the original integer program. Finally we note that in this approach the effect of the $\boldsymbol{U}$ matrix cannot be included. This limitation is discussed below. With these changes, $\boldsymbol{P}$ can be written as

$$\boldsymbol{P}^{'}: \max_{\boldsymbol{w}_{TB},\boldsymbol{S},P_x} log_2 det\left(\boldsymbol{I}_{N_{CR}} + P_x \boldsymbol{S}\boldsymbol{H}_{PC}\boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H\boldsymbol{H}_{PC}^H\boldsymbol{S}^H\right)$$
$$s.t.\quad 0 \leq (\boldsymbol{S})_{ii} \leq 1,\ i = 1,\cdots,N_{CR}$$
$$tr(\boldsymbol{Q}_{PT}) \leq P_{PT} \Rightarrow tr(\boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H) \leq \frac{P_{PT}}{P_x}$$
$$tr(\boldsymbol{S}) = K$$
$$P_x \boldsymbol{h}_{PP,j}\boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H\boldsymbol{h}_{PP,j}^H \leq P_j,\ j = 1,\cdots,N_{PU} \quad (8)$$

Note that the problem $\boldsymbol{P}^{'}$ is still non-convex, due to non-concavity of the objective function. Thus, we seek a convex approximation (CA) to this problem.

Proposition 1: With two of the three variables known, the cost function in the problem $\boldsymbol{P}^{'}$ is concave in the third one and this renders the problem convex in this variable.

Proof: Please refer to Appendix A.

Thus, to solve $\boldsymbol{P}^{'}$, we initialize P_x, $\boldsymbol{w_{TB}}$ and optimize over $\boldsymbol{S}$. Then using the optimum $\boldsymbol{S}$ and the initial value for P_x, optimum TB weight vector is chosen and ultimately, with $\boldsymbol{S}$ and $\boldsymbol{w_{TB}}$ known, optimum value for P_x is obtained. Since elements of $\boldsymbol{S}$ are non-binary, index of chosen antennas are the K-largest diagonal elements of $\boldsymbol{S}$. In Table I, the proposed procedure is summarized.

C. Convergence Analysis

A comment on the convergence of the proposed iterative algorithm is in order here. We argue that during the $(k+1)$ -th iteration $\boldsymbol{S}^{(k+1)} = \text{argmax}_{\boldsymbol{S}}\, \boldsymbol{P}^{'}\left(\boldsymbol{S}, \boldsymbol{w}_{\boldsymbol{TB}}^{(k)}, P_x^{(k)}\right)$ is calculated and we obtain achievable data rate r_1. Then $\boldsymbol{w}_{\boldsymbol{TB}}^{(k+1)} = \text{argmax}_{\boldsymbol{w}_{TB}}\, \boldsymbol{P}^{'}\left(\boldsymbol{w_{TB}}, P_x^{(k)}, \boldsymbol{S}^{(k+1)}\right)$ is calculated, giving rate r_2 . Finally $P_x^{(k+1)} = \text{argmax}_{P_x}\, \boldsymbol{P}^{'}\left(P_x, \boldsymbol{S}^{(k+1)}, \boldsymbol{w}_{\boldsymbol{TB}}^{(k+1)}\right)$is evaluated and the corresponding achievable data rate r_3. Since $r_1 \leq r_2 \leq r_3$ forms a monotonically increasing sequence which is bounded above (due to transmit power constraint) we conclude that the sequence of achievable data rates converges to a limit.

Our simulations indicate that iterating 6 times is almost sufficient to attain an optimum value of $\boldsymbol{P}^{'}$.

TABLE I. PROPOSED ITERATIVE METHOD TO SOLVE $\boldsymbol{P}^{'}$

Step 1-Initializations:
• **Select an initial value for P_x and P_{PT} and an initial w_{TB}**
Step 2-Calculation step:
• **Solve the convex optimization problem and find S** • **Find the optimum w_{TB} that maximizes achievable rates.** • **Solve the problem, knowing S and w_{TB}, to find the optimum P_x**
Step 3-Iterative step:
• **Repeat steps 1 and 2 until convergence. The achievable data rate is the average of results at each SNR.** • **Using the optimum value of P_x, find the SER**

Since the problem $\boldsymbol{P}^{'}$ is not convex in nature, the maximum achievable rates obtained from the proposed iterative rate may not be globally optimum. However, results suggest that the values obtained are robust and are globally optimal most of the time for the parameters and scenario discussed in section II.

IV. Second Phase: Power Allocation for Cooperating CR TXs

In the second phase, opportunistically selected CR TXs are granted the opportunity of transmitting their signals to their intended CR RXs. The transmit power allocation process for CR TXs in the second and third phases can be well modeled by non-cooperative games. As a result, the transmit powers of selected CR TXs in the second and also the third time-slots are the Nash Equilibrium (NE) of the power control game of selected CR TXs, denoted by $\boldsymbol{P}_{CR} = \left[P_{CR,1},\cdots,P_{CR,K}\right]$. The NE of the game will also be used in the third phase by cooperating CR TXs to relay the signals of PU BS to PU RX. The utility function and the constraint of this game can be written as

$$U_{CR,i}\left(P_{CR,i}, \boldsymbol{P}_{CR,-i}\right) =$$
$$\alpha(1-\beta)log\left(1 + \frac{\left|h_{CR,ii}\right|^2 P_{CR,i}}{N_0 + I_0 + \sum_{j=1,j\neq i}^{K}\left|h_{CR,ij}\right|^2 P_{CR,j}}\right) - c_{CR,i} \quad (9)$$
$$P_{CR,i} \leq P_{CR,max} \quad (10)$$

for all $i \in \{1,\cdots,K\}$, where I_0 is the interference seen at CR TXs; N_0 presents the single-sided power spectral density of the white Gaussian noise at CR TXs; $\boldsymbol{P}_{CR,-i}$ denotes the transmit power of CR users, excluding the CR user i; $P_{CR,max}$ represents the maximum transmit power of each CR TX i; $c_{CR,i}$ covers all the expenses which CR users must pay for their transmission in the totally shared channel. Actually, $c_{CR,i}$ is used to alleviate the inefficient Nash Equilibrium of the game which is due to selfish behavior of the CR users. The pricing can be linear or nonlinear. In this work, we propose to use nonlinear pricing method which has been proved to better results than its linear counterpart [13]. Therefore $c_{CR,i}$ is given by [13]

$$c_{CR,i}\left(P_{CR,i}\right) = \lambda exp\left\{\frac{\delta\left(\sum_{j=1,j\neq i}^{K}\alpha P_{CR,i}h_{CR,ij} - Q_{max}\right)}{Q_{max}}\right\} \quad (11)$$

in which Q_{max} specifies the maximum possible interference which CR TXs are allowed to induce on the PU system, δ represents the degree of negative effect of CR TXs on each other and λ is the nonlinear pricing factor. Ultimately, we have the following non-cooperative power control game **G**:

$$\boldsymbol{G}: \max_{P_{CR,i}}\ U_{CR,i}\left(P_{CR,i}, \boldsymbol{P}_{CR,-i}\right) \quad (12)$$
$$P_{CR,i} \leq P_{CR,max}, \forall i \in \{1,\cdots,K\} \quad (13)$$

Regarding the existence of the Nash Equilibrium (NE) of the game G, based on the fixed point theorem, the following theorem has been shown [14]

Theorem 1: A game $(S,(A_i),(u_i))$ (S indicates a finite set of players, A_i denotes a set of actions for each player i and u_i stands for the utility function for

i-th player) has a Nash equilibrium if, for all $i \in S$, the action set A_i of player i is a nonempty compact convex subset of a Euclidian space, and the payoff function u_i is continuous and quasi-concave on A_i.

Proof: Please refer to [14].

Proposition 2: The game $\boldsymbol{G}$ has a unique NE point.

Proof: Please refer to Appendix B.

Proposition 3: The NE of the power control game $\boldsymbol{G}$ of the opportunistically selected CR TXs in the second phase can be obtained in an iterative manner and is the fixed-point of the following equation:

$$P_{CR,i}(n+1) = \left[A \left(\frac{C + \sum_{j=1, j \neq i}^{K} |h_{CR,ij}|^2 P_{CR,j}(n)}{|h_{CR,ii}|^2} + B \right) \right]_0^{P_{CR,max}} \quad (14)$$

for all $i \in \{1, \cdots, K\}$, where $P_{CR,i}(n)$ is the transmit power of the i-th cooperating CR TX in the n-th iteration, $A = \frac{\lambda}{\alpha(1-\beta)}$ and $B = \frac{(1-\beta)Q_{max}}{\lambda \sum_{j=1, j \neq i}^{K} |h_{CR,ij}|^2}$. $[a]_c^b$ denotes the Euclidean projection of a onto the interval $[c, b]$ i.e. $[a]_c^b = c$ if $a < c$, $[a]_c^b = a$ if $c \leq a < b$ and $[a]_c^b = b$ if $a > b$, and therefore the transmit power of selected CR users has been limited to $[0, P_{CR,max}]$.

Proof: Please refer to Appendix C.

Since (14) represents a fixed-point equation, the optimal NE can be determined by iterative methods [18]. We propose two iterative methods to determine the optimal transmit power of cooperating CR TXs. In this way, Jacobi and Gauss-Seidel schemes, are employed to solve (14). The proposed algorithms are represented in Table II.

V. Third Phase: Zero-Frocing Beamforming

In the third phase, the cooperating CR TXs forward the signals of PU BS to PU RX. Therefore, other PUs are vulnerable to interference caused by the cooperation of selected CR TXs with PU BS. We propose to use zero-forcing beamforming (ZFBF) in virtual antenna array which is constituted by the cooperating CR TXs [15]. The transmitted signal of PU BS, intended to cooperating CR TXs, is denoted by $\boldsymbol{x}$ ($\boldsymbol{x} \in \mathbb{C}^{M \times 1}$). The received signal at CR TXs is multiplied to ZFBF weight vector, $\boldsymbol{w}_{ZFBF}$, and is forwarded to PU RX, after being decoded at selected CR TXs. The received signal in PU RX is given by

$$x_{PR} = \boldsymbol{h}_{CP}^{(1)} \boldsymbol{w}_{ZFBF} \hat{x} + z_{PR} \quad (15)$$

where $\boldsymbol{h}_{CP}^{(1)}$ contains the channel vector from cooperating CR TXs to PU RX. The ZFBF weight vector must be calculated such that the SINR in PU RX is maximized. Meanwhile, the interference induced on other PUs, due to transmission of cooperating CR TXs should be removed. Therefore, the following problem should be solved:

$$\begin{aligned} & max_{\boldsymbol{w}_{ZFBF}} \quad \left| \boldsymbol{h}_{CP}^{(1)} \boldsymbol{w}_{ZFBF} \right| \\ & s.t. \quad \left| \boldsymbol{h}_{CP}^{(n)} \boldsymbol{w}_{ZFBF} \right| = 0, \ \forall n \in \{2, \ldots, N_{PU}\} \\ & \qquad \|\boldsymbol{w}_{ZFBF}\| = 1 \end{aligned} \quad (16)$$

where $\boldsymbol{h}_{CP}^{(n)}$, for $n \in \{2, \ldots, N_{PU}\}$, denotes the channel vector from the cooperating CR TXs to the n-th PU. In the sequel, using the orthogonal projection method, we attempt to solve the above-mentioned problem.

Theorem 2: The optimal ZF beamforming weight vector which maximizes $\left| \boldsymbol{h}_{CP}^{(1)} \boldsymbol{w}_{ZFBF} \right|$ and satisfies the constraints given in (16) is the orthogonal projection of $\boldsymbol{h}_{CP}^{(1)}$ onto the orthogonal complementary $\gamma^{\perp}$ of the subspace $\gamma = span\left\{ \boldsymbol{h}_{CP}^{(2)}, \cdots, \boldsymbol{h}_{CP}^{(N_{PU})} \right\}$.

Proof: The channel vector $\boldsymbol{h}_{CP}^{(1)} \in \mathbb{C}^{1 \times K}$, can be written as $\boldsymbol{h}_{CP}^{(1)} = a_1 \boldsymbol{e}_1 + \cdots + a_K \boldsymbol{e}_K$, using the K basis vectors $\{\boldsymbol{e}_1, \cdots, \boldsymbol{e}_K\}$ (where $a_i \in \mathcal{CN}(0,1)$, $i = 1, \cdots, K$ [16]. Hence, the matrix $\boldsymbol{U} = [\boldsymbol{e}_1, \cdots, \boldsymbol{e}_K] = \boldsymbol{I}_K$, is $K \times K$ identity matrix. Then, we consider the set of basis vectors $\{\boldsymbol{e}_1', \cdots, \boldsymbol{e}_{N_{PU}}'\}$, which is the orthogonal basis for the subspace $\gamma = span\left\{ \boldsymbol{h}_{CP}^{(2)}, \cdots, \boldsymbol{h}_{CP}^{(N_{PU})} \right\}$. Actually, γ is a $N_{PU} - 1$dimensional subspace, for the reason that the probability of the realizations of the independent and continuous random vectors $\boldsymbol{h}_{CP}^{(2)}, \cdots, \boldsymbol{h}_{CP}^{(N_{PU})}$ being interrelated is very much small and thus can be ignored. If $\{\boldsymbol{e}_{N_{PU}+1}', \cdots, \boldsymbol{e}_K'\}$ is an orthogonal basis for $\gamma^{\perp}$, the orthogonal set $\{\boldsymbol{e}_1', \cdots, \boldsymbol{e}_{N_{PU}}'\} \cup \{\boldsymbol{e}_{N_{PU}+1}', \cdots, \boldsymbol{e}_K'\}$ is another orthogonal basis for $\gamma^{\perp}$. Similarly, $\boldsymbol{h}_{CP}^{(1)}$ can be represented by $\boldsymbol{h}_{CP}^{(1)} = a_1' \boldsymbol{e}_1' + \cdots + a_K' \boldsymbol{e}_K'$. Clearly, $\boldsymbol{L} = [\boldsymbol{e}_1', \cdots, \boldsymbol{e}_K']$ is also a unitary matrix. Moreover, by matrix manipulation we have

$$[a_1', \cdots, a_K']^T = \boldsymbol{L}^H [a_1, \cdots, a_K]^T \quad (17)$$

Since the random matrix $\boldsymbol{L}^H$ is unitary and independent with $\boldsymbol{h}_{CP}^{(1)}$, $[a_1, \cdots, a_K]$ has the same distribution as $[a_1', \cdots, a_K']$, from $(A.22)$ in [17]. As a result, $a_i' \in \mathcal{CN}(0,1)$, $i = 1, \cdots, K$. The ZFBF weight vector $\boldsymbol{w}_{ZFBF}$ is orthogonal to each $\boldsymbol{h}_{CP}^{(i)}$ ($\forall i \in S_{PU}$ and $i \geq 2$). Hence, it is perpendicular to each vector in γ, and belongs to $\gamma^{\perp}$. In order to maximize $\left| \boldsymbol{h}_{CP}^{(1)} \boldsymbol{w}_{ZFBF} \right|$ in (16), we need to find the vector $\boldsymbol{w}_{ZFBF} \in \gamma^{\perp}$ which is closest to $\boldsymbol{h}_{CP}^{(1)}$. From Closest Point Theorem, $\boldsymbol{w}_{ZFBF,opt}$, is the orthogonal projection of $\boldsymbol{h}_{CP}^{(1)}$ onto the subspace $\gamma^{\perp}$ and we have achieved what we desire. The unit-length constraint in (16) makes such $\boldsymbol{w}_{ZFBF,opt}$ unique.

The achievable data rate between PU BS and PU RX can be written as

$$R_{PP} = min\{(1-\alpha)R_{PC}, \alpha\beta R_{CP}\} \quad (18)$$

where R_{CP} denotes the achievable rate in transmission from opportunistically selected CR users to PU RX.

A. Interactions between desired PU link and CR TXs

The interactions between CR TXs and desired primary link can be modeled by Stackelberg Games [14], where PU BS is the leader and CR TXs are followers. The leader of the game takes its strategy, i.e. choosing appropriate α and β, in a way that maximizes its data rate. Since β only exists in $\alpha\beta R_{CP}$ in Equation (18), its optimal value is determined by solving the following problem

$$\beta_{opt} = \text{argmax}_{\beta\in[0,1]}\ \beta R_{CP} \tag{19}$$

where $R_{CP} = log_2\left(1 + \frac{\sum_{i=1}^{K}\left|\boldsymbol{h}_{CP}^{(1)}(i)\boldsymbol{w}(i)\right|^2 P_{CR,i}}{N_0+I_0}\right)$. The optimal value of α can be written as

$$\alpha_{opt} = \frac{1}{1+\frac{\beta_{opt}R_{CP}}{R_{PC}}} \tag{20}$$

Ultimately, the optimum achievable rate from PU BS to PU RX is given by

$$R_P = \frac{\beta_{opt}R_{CP}R_{PC}}{R_{PC}+\beta_{opt}R_{CP}} \tag{21}$$

where $R_{PC} = log_2\left(1 + \frac{P_t\sum_{i=1}^{K}\|\boldsymbol{H}_{PC}(i,:)\|^2}{N_0+I_0}\right)$ represents the data rate from PU BS to selected CR users.

It is noteworthy that the Golden Section search method can be recruited to solve (19) and determine the optimal value of β. The method of Golden Search was presented in [19]. Here, we aim to provide a brief description. First of all, two points a_1 and b_1 are determined using

$$\begin{cases} a_1 = a_0 + \rho(b_0 - a_0) \\ b_1 = a_0 + (1-\rho)(b_0 - a_0) \end{cases} \tag{22}$$

where $\rho < \frac{1}{2}$ and a_0 and b_0 are equal to 0 and 1, accordingly. If $a_1R_{CP}(a_1) < b_1R_{CP}(b_1)$, then the maximizer of βR_{CP} is located in the interval $[a_1, b_0]$. Otherwise it can be deduced that the maximizer point is in the interval $[a_0, b_1]$. The search for optimal amount is continued by limiting the search section and new search points, calculated in the appropriate interval, using (22).

VI. PERFORMANCE EVALUATION

In this section, a simulation setup is explained and the performance of the proposed algorithms is evaluated using simulations.

A. Simulation Assumptions

Simulations are performed based on the following assumptions, otherwise stated:

- There are 4 PUs in the system, including PU BS, PU RX and two other PUs and PU BS is equipped with 3 antennas.
- CVX package is used along with MATLAB to run simulations [20].
- Achievable rates are determined by averaging over the results obtained from 1000 i.i.d. channels realizations.
- We introduce a parameter, τ, which controls the interference threshold at other PUs in first phase. τ is chosen so that the tolerable amount of interference at other PUs in first phase is a fraction of their received SNR, i.e. $P_i = \tau SNR_{PU}$.
- The codebook matrix for three antennas at PU BS and three limited feedback bits is given by [9].

TABLE II. PROPOSED ALGORITHMS TO DETERMINE THE OPTIMAL TRANSMIT POWER OF COOPERATING CR TXS AND THE OPTIMAL VALUE OF β

Initializations:

1) **Choose an initial value for $P_{CR,i}(0)$ (for all i) and $\beta(0)$**
2) **Choose ε (Stooping Condition)**

Stopping Condition:
If for all $i\epsilon\{1,\cdots,K\}$,
$|P_{CR,i}(n+1) - P_{CR,i}(n)| < \varepsilon$ **and** $|\beta(n+1) - \beta(n)| < \varepsilon$
Then STOP
Else Continue

Iterative Step: Iteration count: n=1;

While Stopping Condition is not satisfied do
For all $i\epsilon\{1,\cdots,K\}$ Update $P_{CR,i}$, using one of following schemes :

Jacobi Scheme

$$P_{CR,i}(n+1) = \left[A\left(\frac{C+\sum_{j=1, j\neq i}^{K}|h_{CR,ij}|^2 P_{CR,j}(n)}{|h_{CR,ii}|^2} + B\right)\right]_0^{P_{CR,max}}$$

$$A = \frac{\lambda}{\alpha(1-\beta)},\ B = \frac{(1-\beta)Q_{max}}{\lambda\sum_{j=1,j\neq i}^{K}|h_{CR,ij}|^2},\ C = N_0 + I_0, i = 1,\cdots,K$$

Gauss-Seidel Scheme:

$$P_{CR,i}(n+1) = \left[A\left(\frac{C+D+E}{|h_{CR,ij}|^2} + B\right)\right]_0^{P_{CR,max}}$$

$$D = \sum_{j<i}^{K}|h_{CR,ij}|^2 P_{CR,j}(n+1),\qquad E = \sum_{j>i}^{K}|h_{CR,ij}|^2 P_{CR,j}(n)$$

for all $i\epsilon\{1,\cdots,K\}$

Update β using Golden Section Search method

End For
n=n+1
End While

$$\boldsymbol{K} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}e^{\frac{2\pi j}{3}} & \frac{1}{\sqrt{2}}e^{\frac{4\pi j}{3}} & \frac{1}{\sqrt{2}}e^{\frac{4\pi j}{3}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}e^{\frac{4\pi j}{3}} & 0 & 0 & \frac{1}{\sqrt{2}}e^{\frac{2\pi j}{3}} & \frac{1}{\sqrt{2}}e^{\frac{4\pi j}{3}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}e^{\frac{4\pi j}{3}} & \frac{1}{\sqrt{2}}e^{\frac{2\pi j}{3}} & 0 & \frac{1}{\sqrt{2}}e^{\frac{2\pi j}{3}} \end{bmatrix}$$

B. Opportunistic Cooperation

It can be inferred from Fig. 2 that as the set of available CR users grows, achievable data rates increase due to multiuser diversity. Therefore opportunistic cooperation results in an increase in data rates of the proposed system. Moreover, increasing the interference threshold at other PUs in first phase, τ, is a very effective way for increasing the achievable rate in the desired link, as proved by simulations in Fig. 2.

C. Zero-Forcing Beamforming

As shown in Fig. 3, in addition to removing the interference, increasing the achievable data rates is another privilege for the proposed system, due to incorporating ZFBF. According to Fig. 3, there are only 2 CR user and thus no user selection occurs, but ZFBF is done at cooperating CR users, data rates are larger than when 2 cooperating CR users are selected among 5 available ones, but ZFBF is not implemented at cooperating CR users.

D. Optimum values for α

As shown in Fig. 4, as SNR increases, the optimum value of α also increases. This can be attributed to more inclination of the PU BS to cooperate with CR users in high SNRs. Moreover, more selected CR users provides the system with more utilization of space and also cooperation. Therefore, less time is required to be allocated to cooperation phase. Another important point is that as other PUs increase their tolerable amount of interference in the first phase, PU BS is more stimulated to recruit CR users as cooperative relay. Finally, opportunistic cooperation persuades the PU BS to use CR users as relay, as shown in Fig. 4.

VII. CONCLUSIONS

In this paper the issue of dynamic spectrum leasing with an opportunistic cooperative approach was investigated. The core aim is to maximize the achievable data rates in the desired PU link, taking advantage of the cooperation of the opportunistically selected CR users. Moreover, the beamforming in the virtual antenna array constituted by selected CR users is recruited. The main difference of the proposed approach with related works is to consider multiple PUs and guarantee their QoS using the TB at PU BS and also the distributed beamforming at cooperating CR users. Also, the cooperating CR users are selected among many available CR users, in order to take advantage of the multiuser diversity.

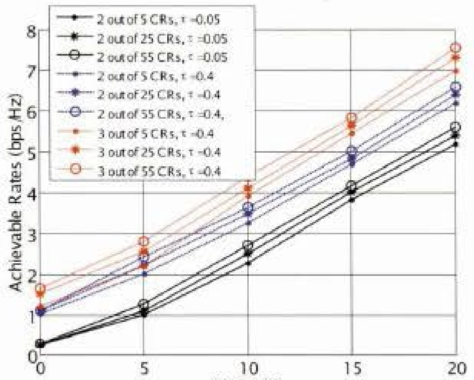

Fig. 2. Achievable rates versus SNR for different values of available CR users and τ

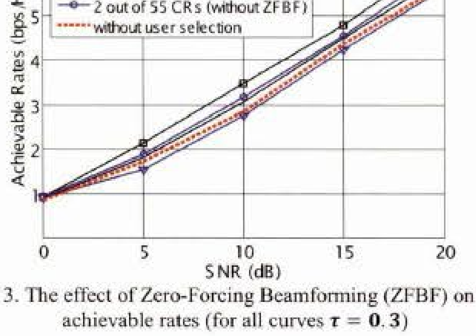

Fig. 3. The effect of Zero-Forcing Beamforming (ZFBF) on the achievable rates (for all curves $\tau = 0.3$)

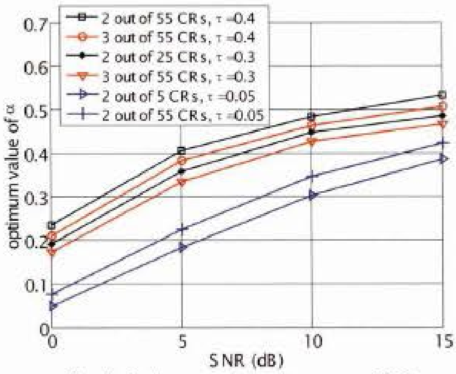

Fig. 4. Optimum value of $\boldsymbol{\alpha}$ versus SNR

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APPENDIX A: PROOF OF PROPOSITION 1

Three different cases must be investigated:

1) When $\boldsymbol{S}$ and P_x are known and $\boldsymbol{w}_{TB}$ is unknown.
2) When $\boldsymbol{w}_{TB}$ and P_x are known and $\boldsymbol{S}$ must be specified.
3) When $\boldsymbol{w}_{TB}$ and $\boldsymbol{S}$ are known and P_x must be found.

Case 1: When $\boldsymbol{S}$ and P_x are known and $\boldsymbol{w}_{TB}$ is unknown.

In this case, the objective function can be written as

$$g(\boldsymbol{W}_{TB}) = log_2 det\left(\boldsymbol{I}_{N_{CR}} + P_x \boldsymbol{H}_{PC} \boldsymbol{W}_{TB} \boldsymbol{H}_{PC}^H \boldsymbol{S}^H\right) \quad (23)$$

where $\boldsymbol{W}_{TB} = \boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H$ and considering the constraints, there is no difference between maximizing $\boldsymbol{w}_{TB}$ and $\boldsymbol{W}_{TB}$. According to [11], $g(\boldsymbol{W}_{TB})$ is concave, if and only if $f(t) = g\left(t\boldsymbol{W}_{TB}^{(1)} + (1-t)\boldsymbol{W}_{TB}^{(2)}\right)$ is concave for every feasible $\boldsymbol{W}_{TB}^{(1)}$, $\boldsymbol{W}_{TB}^{(2)}$ and $0 \le t \le 1$. Therefore $f(t)$ can be written as

$$f(t) = log_2 det\left(\boldsymbol{I}_{N_{CR}} + P_x \boldsymbol{S}\boldsymbol{H}_{PC}\left(\boldsymbol{W}_{TB}^{(2)} + t\left(\boldsymbol{W}_{TB}^{(1)} - \boldsymbol{W}_{TB}^{(2)}\right)\right)\boldsymbol{H}_{PC}^H \boldsymbol{S}^H\right)$$
$$= log_2 det(\boldsymbol{A} + t\boldsymbol{B}) \quad (24)$$

where $\boldsymbol{A} = \boldsymbol{I}_{N_{CR}} + P_x \boldsymbol{S}\boldsymbol{H}_{PC}\boldsymbol{W}_{TB}^{(2)}\boldsymbol{H}_{PC}^H\boldsymbol{S}^H$ and $\boldsymbol{B} = P_x\boldsymbol{S}\boldsymbol{H}_{PC}\left(\boldsymbol{W}_{TB}^{(1)} - \boldsymbol{W}_{TB}^{(2)}\right)\boldsymbol{H}_{PC}^H\boldsymbol{S}^H$. $f(t)$ can be further manipulated as

$$f(t) = log_2 det\left(\boldsymbol{A}^{\frac{1}{2}}\left(\boldsymbol{I}_{N_{CR}} + t\boldsymbol{A}^{\frac{-1}{2}}\boldsymbol{B}\boldsymbol{A}^{\frac{-1}{2}}\right)\boldsymbol{A}^{\frac{1}{2}}\right)$$
$$= \sum_{i=1}^{n} log_2(1 + t\lambda_i) + log_2 det\boldsymbol{A} \quad (25)$$

In which $\lambda_1, \cdots, \lambda_n$ denote the eigen-values of $\boldsymbol{A}^{-1/2}\boldsymbol{B}\boldsymbol{A}^{-1/2}$. Hence:

$$\frac{df(t)}{dt} = \frac{1}{ln2}\sum_{i=1}^{n}\frac{\lambda_i}{1+t\lambda_i} \quad (26)$$

$$\frac{d^2f(t)}{dt^2} = \frac{-1}{ln2}\sum_{i=1}^{n}\frac{\lambda_i^2}{(1+t\lambda_i)^2} \quad (27)$$

Negativity of $\frac{d^2f(t)}{dt^2}$ results in concavity of $f(t)$. Then the concavity of $g(\boldsymbol{W})$ is concluded.

Case 2: P_x and $\boldsymbol{w}_{TB}$ known, $\boldsymbol{S}$ unknown.

First of all, the objective function must be manipulated as

$$log_2det\left(\boldsymbol{I}_{N_{CR}} + P_x\boldsymbol{H}_{PC}\boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H\boldsymbol{H}_{PC}^H\boldsymbol{S}^H\right) = log_2det(1 + P_x\boldsymbol{w}_{TB}^H\boldsymbol{H}_{PC}^H\boldsymbol{S}^H\boldsymbol{S}\boldsymbol{H}_{PC}\boldsymbol{w}_{TB}) \quad (28)$$

And by introducing $\boldsymbol{V} = \boldsymbol{S}^H\boldsymbol{S}$, the concavity of the objective function is proved in a similar method employed in case 1.

Case 3: $\boldsymbol{S}$ and $\boldsymbol{w}_{TB}$ known, P_x unknown.

In this case, the objective function can be written as

$$g(P_x) = log_2det\left(\boldsymbol{I}_{N_{CR}} + P_x\boldsymbol{A}\right) \quad (29)$$

where $\boldsymbol{A} = \boldsymbol{S}\boldsymbol{H}_{PC}\boldsymbol{w}_{TB}\boldsymbol{w}_{TB}^H\boldsymbol{H}_{PC}^H\boldsymbol{S}^H$. Once again, $f(t)$is defined as $g\left(tP_x^{(1)} + (1-t)P_x^{(2)}\right)$ where $P_x^{(1)}$ and $P_x^{(2)}$are any feasible amounts for P_x and $0 \le t \le 1$:

$$f(t) = log_2det\left(\boldsymbol{I}_M + \left(P_x^{(2)} + t\left(P_x^{(1)} - P_x^{(2)}\right)\right)\boldsymbol{A}\right) = log_2det(\boldsymbol{B} + t\boldsymbol{C}) \quad (30)$$

where $\boldsymbol{B} = \boldsymbol{I}_{N_{CR}} + P_x^{(2)}\boldsymbol{A}$ and $\boldsymbol{C} = \left(P_x^{(1)} - P_x^{(2)}\right)\boldsymbol{A}$ and the concavity of the $g(P_x)$ is derived in a similar manner as employed in previous cases.

APPENDIX B: PROOF OF PROPOSITION 2

The convexity of the power constraint in (10) is clear. Hence we prove the quasi-concavity of the utility function in (9). The second derivative of the utility function can easily be written as

$$\frac{\partial^2 U_{CR,i}\left(P_{CR,i}, \boldsymbol{P}_{CR,-i}\right)}{\partial P_{CR,i}^2} = \frac{-\alpha(1-\beta)\left|h_{CR,ii}\right|^4}{N_0 + I_0 + \sum_{j=1,j\neq i}^K\left|h_{CR,ij}\right|^2 P_{CR,j} + \left|h_{CR,ii}\right|^2 P_{CR,i}} - \frac{\lambda\delta\sum_{j=1,j\neq i}^K\alpha\left|h_{CR,ij}\right|^2}{Q_{max}} exp\left\{\frac{\delta\left(\sum_{j=1,j\neq i}^K\alpha P_{CR,i}h_{CR,ij} - Q_{max}\right)}{Q_{max}}\right\} \le 0 \quad (31)$$

Therefore the utility function is quasi-concave.

APPENDIX C: PROOF OF PROPOSITION 3

To verify this proposition and find the NE, it is enough to set the derivative of the utility function of selected CR users to zero:

$$\frac{\partial U_{CR,i}\left(P_{CR,i}, \boldsymbol{P}_{CR,-i}\right)}{\partial P_{CR,i}} = \frac{-\alpha(1-\beta)\left|h_{CR,ii}\right|^2}{N_0 + I_0 + \sum_{j=1,j\neq i}^K\left|h_{CR,ij}\right|^2 P_{CR,j} + \left|h_{CR,ii}\right|^2 P_{CR,i}} - \frac{\delta\sum_{j=1,j\neq i}^K\alpha\left|h_{CR,ij}\right|^2}{Q_{max}} c_{CR,i} \quad (32)$$

Let $C_1 = \frac{\delta\sum_{j=1,j\neq i}^K\alpha\left|h_{CR,ij}\right|^2}{Q_{max}}$ and $C_2 = \frac{N_0 + I_0 + \sum_{j=1,j\neq i}^K\left|h_{CR,ij}\right|^2 P_{CR,j}}{\alpha(1-\beta)\left|h_{CR,ii}\right|^2}$. By assuming $\frac{-\delta\left(\sum_{j=1,j\neq i}^K\left|h_{CR,ij}\right|^2 P_{CR,j} - Q_{max}\right)}{Q_{max}}$ to be small and also setting (32) equal to zero we obtain

$$\frac{1}{\alpha(1-\beta)}P_{CR,i} + \frac{1}{\lambda C_1}\frac{-\delta\left(\sum_{j=1,j\neq i}^K\left|h_{CR,ij}\right|^2 P_{CR,j} - Q_{max}\right)}{Q_{max}} + C_2 = 0 \quad (33)$$

Ultimately, by applying the maximum transmit power constraint the NE is acquired as in (14).

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