Joint Antenna Selection and Adaptive Modulation in MIMO Systems in the Presence of CSI Imperfection

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Abstract- This paper presents a strategy to combine the antenna selection technique and the adaptive modulation method for multiple input multiple output (MIMO) systems. The MIMO architecture significantly improves the performance of wireless systems at the expense of a radio frequency (RF) hardware complexity. However, the advantages of MIMO systems can be mainly maintained with simple hardware using antenna selection technique. Moreover, in order to improve the efficiency, the adaptive modulation method can be implemented. As a practical system design limitation, the perfect channel state information (CSI) may not be achievable. Accordingly, the effect of imperfect channel state information (ICSI) in the proposed systems has been investigated.

Keywords-MIMO; antenna selection; adaptive modulation; CSI imperfection; spectral efficiency; bit error rate

I. INTRODUCTION

Multiple Input Multiple Output (MIMO) technology can significantly improve the performance of systems. This performance can be the diversity gain, spatial multiplexing gain or a trade-off between these two [1]. These performance improvements come at the expense of the hardware complexity. Antenna selection is proposed to capture the advantages of MIMO systems while keeping low complexity hardware. In [2], the authors proposed a method for optimal selection of transmit antennas based on the capacity maximization. This technique is also studied for the Alamouti’s space-time coding [3],[4]. A promising approach for the fast antenna subset
selection was proposed by Gorokhov [5]. In [6], computational lower-complexity algorithm was proposed. In [7-11], authors studied the different issues in antenna selection technique. On the other hand, the adaptive modulation (AM) method is a technique to increase the average spectral efficiency (ASE) in wireless systems. In [12-15], authors studied the adaptive modulation technique which can adapt the transmitter parameters as constellation size and transmit power to maximize the ASE. The MIMO system performance with adaptive modulation in presence of CSI imperfection was provided in [16]. In this paper, a design that combines antenna subset selection and adaptive modulation method has been proposed. The proposed scheme maximizes the ASE of a MIMO system for a given BER constraint and constant power considering imperfect channel state information.

The paper has been organized as follows: Section II introduces the system model. Section III describes the antenna selection algorithm. In the section IV, the adaptive modulation has been implemented for the proposed system. It is followed by the introducing the joint adaptive antenna subset selection and modulation mode selection algorithm in ICSI state. Finally, the simulation results are presented and the conclusions are drawn.

II. SYSTEM MODEL

A MIMO communication system consisting of \( N_t \) transmit antennas and \( N_r \) receive antennas is presented in figure 1. The channel model is considered Rayleigh flat fading model.

![MIMO system model](image1)

Initially, we assume perfect CSI is available at both transmitter and receiver. Later, we investigate the effect of CSI imperfection. The matrix channel is represented by an \( N_t \times N_r \) matrix. The entries of \( H \) are independent from each other and \( H_{ij} \sim \text{CN}(0,1) \). The channel input is an \( 1 \times N_t \) column vector \( x \), and the channel output is represented by \( 1 \times N_r \) column vectors \( y \), respectively. The entries of noise matrix are also assumed to be independent and \( N_i \sim \text{CN}(0,\sigma^2) \). The channel input/output equation can be written as:

\[
y = xH + n
\]

Applying a Singular Value Decomposition (SVD) to \( H \), it can be expressed as:

\[
H = U\Sigma V^H
\]

where \( \Sigma \) is an \( N_t \times N_r \) nonnegative and diagonal matrix with the singular values of \( H \) and main diagonal elements \( \lambda_i \). Note that \( \{\lambda_i\} \) is the eigenvalues of \( H^H \). For convenience, the eigenvalue vector \( \lambda \) is defined as \( \lambda \triangleq [\lambda_1, ..., \lambda_r]^T \).

U=\( [u_1, ..., u_{N_t}] \) and \( V=[v_1, ..., v_{N_r}] \) are \( N_t \times N_t \) and \( N_r \times N_r \) unitary matrices with left and right singular vectors of \( H \) as their columns, respectively. Inserting (2) in (1), the following can be reached:

\[
y' = xD + \tilde{n}
\]

Where

\[
\tilde{y} \triangleq yV^H \quad \hat{x} \triangleq xU \quad \tilde{n} \triangleq nV^H
\]

Since \( U \) and \( V \) are unitary matrices, it is important to note that the powers of \( x \) and \( \hat{x} \) are the same, as well as \( y \) and \( \tilde{y} \), \( n \) and \( \tilde{n} \). From (3), it can be observed that the channel matrix \( H \) has been decomposed into \( r \) parallel eigen-subchannels since \( D \) is diagonal. The equivalent channel input and output are \( \hat{x} \) and \( \tilde{y} \), respectively. The subchannel power gains are represented by \( \lambda \), which constitute a random process due to the randomness of the channel entries of \( H \). According to (3), if the channel matrix \( H \) or the vectors \( (U,V,\lambda) \) are perfectly available at both the transmitter and receiver, the equivalent system model can be shown in Fig 2.
III. ANTENNA SELECTION ALGORITHM

Antenna selection is a promising way of capturing a large portion of the channel capacity in MIMO systems at reduced hardware cost and computational complexity. In order to make antenna selection, instead of computing the capacity for all possible combinations of selected antennas (as in the optimal selection procedure), several suboptimal selection algorithms can be used [5], [6]. In this paper, the suggested algorithm in [6] has been applied. The algorithm starts with one antenna per step. In each step, the objective was to select one more antenna leading to the highest increase in the capacity. In the $n$th step of the algorithm, the channel matrix corresponding to the $n$ selected receive antennas was denoted by $H_n$. The $H_n$ matrix contains $n$ rows of $H$ matrix (in the same order as $H$), corresponding to the selected antennas. The $j$th row of $H$ matrix is its Hermitian transpose by $h_j$. In the $(n+1)$th step, the receive antenna corresponding to the $J$th row of $H$ is selected.

\begin{equation}
C(H_{n+1})=C(H_n)+\log_2 \det(1+\frac{\rho}{N_t} \alpha_{j,n})
\end{equation}

where, for any $j$, the value of $\alpha_{j,n}$ represents the contribution of the $j$th receive antenna to the capacity expression under the log function if the antenna is selected in the $(n+1)$ step of the algorithm. Finding $J$ that maximizes $C(H_{n+1})$ in (4) is equivalent to obtaining:

\begin{equation}
J=\arg\max_j \alpha_{j,n}
\end{equation}

that

\begin{equation}
\alpha_{j,n} \triangleq h_j B_n h_j^H
\end{equation}

and

\begin{equation}
B_n \triangleq \left(I_{N_t} + \frac{\rho}{N_t} H_n H_n^H\right)^{-1}
\end{equation}

IV. ADAPTIVE MODULATION

Adaptive modulation is a technique to increase the average spectral efficiency in wireless systems. In comparison to fixed modulation order scheme, this technique provides the higher rate and more efficient used in the resources. Adaptive modulation algorithm can be implemented to improve spectral efficiency by adjusting the modulation size depending on the condition of fading channels. In this case, more bits are transmitted when the channel quality is good. Using this technique, we need feedback signal from receiver to transmitter to show the channel state information (CSI). The objectives to use the adaptive modulation over a Rayleigh fading channel with CSI at the transmitter are to maximizing the spectral efficiency at a target BER and minimizing BER at a given throughput.

In the first state, a constant transmit power cutoff threshold was selected. The SNR thresholds are to be solved to maximize spectral efficiency while satisfying certain constraints. The average power constraint is

\begin{equation}
\int_0^\infty S(\gamma) p_{\gamma}(\gamma) \leq S
\end{equation}

where $\gamma_0$ is threshold, and the constant transmit power $S(\gamma)=S$ satisfies

\begin{equation}
S = \frac{1}{\int_{\gamma_0}^\infty p_{\gamma}(\gamma) d\gamma}
\end{equation}

To satisfy the instantaneous BER constraint $BER(\gamma) \leq \bar{BER}$ for all $\gamma$ and to maximize spectral efficiency, the BER constraint must be satisfied at each boundary point $\gamma_j$:

\begin{equation}
BER(\gamma_j) \approx c_1 \exp\left[ \frac{-c_2}{c_3 \lambda_{\gamma_j}} \right] \leq \bar{BER}
\end{equation}

For a constant transmit power, $BER(\gamma) \leq BER(\gamma_j)$ for $\gamma_j \leq \gamma \leq \gamma_{j+1}$, therefore, the optimal rate region boundaries $\gamma_j$ ($1 \leq j \leq N-1$) can be obtained by solving (10) and $c_1$, $c_2$, $c_3$, are positive fixed constant, and $c_4$ is a real constant that is depending on type of modulation and approximation.

V. ANTENNA SELECTION WITH ADAPTIVE MODULATION WITH CSI IMPERFECTION

The system model is shown in Fig 2. This means that there are $r$ SISO subchannels. Because of selecting antenna, the pdf of eigenvalue of $HH^H$ was irregular. To obtain the probability distribution function of $\lambda_i$,

We Assume Wishart distributed the marginal probability density function of the $k$:th largest eigenvalue is given by:

\begin{equation}
p_k(\lambda_k) = Q \int_{\lambda_{k-1}}^{\infty} d\lambda_{k-1} \int_{\lambda_{k-2}}^{\infty} d\lambda_{k-2} \ldots \int_{\lambda_3}^{\infty} d\lambda_2 \int_{\lambda_2}^{\infty} d\lambda_1 \int_0^{\lambda_1} d\lambda_k \ldots \int_0^{\lambda_2} d\lambda_{r+1} p(\lambda_1, ..., \lambda_r)
\end{equation}
If Q is constant, the pdf is normalized to unity, and
\[ p(\lambda_1, ..., \lambda_r) = K_{r,n} \prod_{i=1}^{r} \left( e^{-\lambda_i} \lambda_i^{n-1} \right) \prod_{i=1}^{r} \left( \lambda_i - \lambda_i \right)^2 \] (12)

where
\[ K_{r,n} = \frac{\pi^{r-1}}{\prod_{i=1}^{r} \lambda_i} \]
\[ \prod_{a=1}^{r} (a-i+1) \]

If it is desirable to normalize the marginal pdf to have the property \( \int_0^\infty p_\lambda(\lambda_k) d\lambda_k = 1 \), the normalization constant Q is used. Therefore, the spectral efficiency is given by:
\[ R = \frac{\sum_{i=0}^{r-1} \sum_{j=0}^{m-1} k_j p_i(y_i) \leq y_{i+1}}{m} \]
(13)

So far, we assume perfect CSI is available at both transmitter and receiver. We refer to these systems as ideally designed MIMO AM systems. Now, we investigate the effect of imperfect CSI and evaluate the ASE performance in the presence of CSI imperfection. The imperfect CSI is represented by the channel matrix \( \mathbf{H} \), which is related to the initial channel matrix \( \mathbf{H} \) as,
\[ \mathbf{H} = \mathbf{H} + \sigma_\epsilon \mathbf{X} \]
(14)

where \( \mathbf{X} \) is the CSI imperfection matrix. Note that it is assumed that ideal CSI is known to the transmitter and receiver. The entries of both \( \mathbf{H} \) and \( \mathbf{X} \) are modelled as independent Gaussian variables, furthermore, \( \mathbf{H} \) and \( \mathbf{X} \) are independent of each other. Thus, we can write \( x_{i,j} \sim \mathcal{CN}(0,1) \) and \( \mathbf{H}_{i,j} \sim \mathcal{CN}(0,1) \), where \( \sigma^2 \) reflects the accuracy of the CSI, and \( \rho^2 \triangleq 1 - \sigma^2 \).

Similar to (2), by applying a SVD to \( \mathbf{H} \), we have
\[ \mathbf{H} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H \]
(15)

\[ \mathbf{D} = \text{diag}\left[ \sqrt{\lambda_1}, ..., \sqrt{\lambda_m} \right] \]
where all notations have similar meaning as those in (3). Since the variable rate system is designed under a perfect CSI assumption, it should take the imperfect-H. The equivalent system (3) becomes
\[ \hat{y} = \hat{H}^H (\mathbf{H} \mathbf{V} \mathbf{x} + \mathbf{n}) \]
(16)

The first term shows information-carrying component and other term shows the cross subchannel interferences. This term is treated as interferences and thus the SNR of the \( i \)th subchannel with equal power allocation is defined as
\[ y_j = \frac{\rho^2 \lambda_i \lambda_j + \sigma_n^2}{\sigma^2 \lambda_j + \sigma_n^2} \]
(18)

VI. SIMULATION RESULTS

In this section, the bit error rate (BER) and ASE performances of our system are evaluated by several simulations over a large number of channel realizations. For discrete rate cases it is assumed that different MQAM signal constellation (4QAM,16QAM,64QAM,64QAM) are available. In addition BPSK is used in some cases. i.e., \( k_j = 0 \) with \( SE_0 = 0 \), which means no transmission.

To simplify the implementation, the power is equally distributed to each active transmit antenna (i.e.,uniform power allocation). Moreover, SNR boundaries in Rayleigh fading are computed as follows:
\[ \text{BER}(y_j) = c_1 \exp\left[ \frac{-c_2 y_j}{y_j + \frac{1}{y_j}} \right] \leq \text{BER} \]
(19)

Using MQAM modulation, the constants in (10) are:
\[ c_1 = 0.2, c_2 = 1.5, c_3 = 1, c_4 = 1 \]

Therefore,
\[ \text{BER}(y_j) = 0.2 \exp\left[ \frac{-1.5 y_j}{y_j + \frac{1}{y_j}} \right] \leq \text{BER} \]
(20)

and
\[ y_j = \frac{2}{3} \ln(5\text{BER}_j)(2^k - 1) \]
(21)
For different $k_i$, the different $\gamma_i$ are obtained. The rate adaptation of MQAM is done based on the received SNR as follows:

$$\gamma_i = \frac{P}{m \sigma_{\epsilon}^2}$$

where $\gamma_i$ is received SNR which is compared with SNR threshold. Accordingly, the suitable modulation size is selected.

Since there is $r$ sub-channel, we can exert our algorithm on each sub-channel to find the suitable modulation size for each sub-channel and finally obtain ASE of the system from (19). Moreover, we can compute average BER as the ratio of average number of error bits and average number of bits:

$$\text{BER} = \frac{E \left[ \text{number of error bits per transmission} \right]}{E \left[ \text{number of bits per transmission} \right]} \quad (22)$$

Fig.3 shows the analytical ASE versus SNR with and without CSI imperfection for $N_t=2$, $N_r=5$, $L_r=2$, transmit and receive and selected antenna configuration, respectively. $k_j=1,2,\ldots,6$ bits/symbol set of signal constellations were used, a target BER of $10^{-3}$ in each mode was considered and then the thresholds from (10) were achieved.

As expected, the ASE reduced with increasing the CSI imperfection. For example, if $\sigma_{\epsilon}^2=10^{-3}$ the gap between perfect CSI diagram and diagram with $\sigma_{\epsilon}^2=10^{-3}$ is about 6 dB and this gap is reduced by increasing $\sigma_{\epsilon}^2$. Note that, the spectral efficiency is more affected by imperfect channel estimation in high SNR regions.

Fig.5 shows the analytical ASE versus $\sigma_{\epsilon}^2$ for $N_t=2$, $N_r=5$, $L_r=2$ transmit, receive and selected antenna configurations, respectively. The set of signal constellations $k_i=1,2,4,6$ bits/symbol were used and a target BER of $10^{-3}$ in each mode was considered. In comparison with Fig .3, the results show that by decreasing receive antennas, ASE is also decreasing.
were used and a target BER of $10^{-3}$ in each mode was considered.

![Graph showing Average BER vs. $\sigma^2$ for different SNR](image)

**Fig.6 Average BER versus $\sigma^2$ for different SNR**

We see that in CSI imperfection with $\sigma^2 < 10^{-3}$, the actual BER is almost constant at about $10^{-4}$ and below the target BER. When CSI is perfect or is nearly perfect, the actual instantaneous BER lies below the target BER. As the CSI imperfection gets more severe, in particular $\sigma^2 < 10^{-2}$, the BER becomes very sensitive to the CSI imperfection.

VII. CONCLUSION

A joint antenna selection and adaptive modulation with perfect and imperfect CSI was analyzed and simulated. This system uses the advantages of both systems. The results show the ASE improvement. However, the amount of improvement is very sensitive to CSI imperfection.

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REFERENCES

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