

Joint Power Adaptation and Interference Avoidance for Unlicensed Wireless Systems: A Game Theoretic Approach

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Abstract—In multiuser wireless systems, waveform adaptation is one of the interference management methods in which the users adaptively change their transmitting waveforms in signal space to enhance the system performance subject to some Quality-of-Service (QoS) requirements. In this paper, the uplink scenario of a Multi Carrier Code Division Multiple Access (MC-CDMA) is considered under slow and frequency selective fading. An admissibility test under maximum received power constraint is presented to check that if there exist feasible powers and waveforms for network users to hold the QoS constraint of each user above a predetermined level. In our set-up, a number of licensed (fixed) and unlicensed (agile) users share some orthogonal carriers to send data to a common receiver (or co-located independent receivers), subject to specific target SINRs for users. We analyze this system as a non-cooperative separable game and show the existence of Nash equilibrium point for this joint power control and codeword adaptation game. An iterative distributed algorithm based on better response strategy is presented which at the receivers results in an ensemble of well-known Generalized Welch Bound Equality (GWBE) optimal waveforms and orthogonal waveforms for oversized users in addition to optimal transmission power for each user. We investigate the properties of fixed point and the effect of algorithm on the performance of fixed users' network. Finally the algorithm will be confirmed by the numerical examples.

Keywords-Unlicensed communication; interference avoidance; MC-CDMA; waveform adaptation; transmitter optimization.

I. INTRODUCTION

The increasing demand for personal high data rate wireless applications and the limited spectrum range

have tended to change the policy of licensed spectrum usage, allowing unlicensed systems to transmit on temporary unused licensed frequencies. Spectrum sharing provides the capability to maintain the QoS of

fixed users with avoiding interference to the agile users by coordinating the multiple-access of fixed users as well as allocating communication resources adaptively to the changes of radio environment[1].

For a multiple-access channel, the main limiting factor is the multiple-access interference (MAI).

is an efficient technique by which transmitters in a wireless communication system are optimized against changing patterns of interference subject to required QoS. Centralized algorithms for constructing optimal spreading codes and powers in single cell Synchronous CDMA subject to required SINR for each user is presented in [2, 3], where admissibility region and user capacity of CDMA system are mentioned. A distributed algorithm based on distributed noncooperative game theory under ideal channel (slow and frequency non selective fading channel with unit gain) is proposed [4-7]. Extensive simulations have shown that the algorithm reaches a GWBE ensemble of codewords and powers for users for which the sum of allocated powers among all valid power allocations for the given target SINRs is minimum[3].

High data rate communication systems require modulation techniques that improve the band efficiency and system robustness against fading. Multi-carrier code division multiple access (MC-CDMA) is one of the modulation methods that can be used to accomplish these demands[8]. In MC-CDMA, instead of applying spreading sequences in the time domain, we can apply them in the frequency domain, mapping a different chip of a spreading sequence to an individual OFDM subcarrier[9]. Greedy interference avoidance algorithm is typically applied to uplink MC-CDMA dispersive channel with fixed and equal power for all users[10, 11] where no constraint except unity norm of spreading sequence is considered. It is shown that, greedy interference avoidance monotonically increases sum capacity (or equivalently decrease the total squared correlation) of channel and hence, the codeword ensemble derived from algorithm, maximizes the sum capacity. Recently a game theoretic algorithm based on [4-7, 10, 11] has been proposed to joint transmitter adaptation and power control[12]. In these works all of the users adapt their waveforms in white noise background. An analytical study is performed on the fixed points for mixtures of fixed and agile users in [13] where the power of users are fixed and equal.

In this paper we use the adaptive greedy interference avoidance approach as well as power allocation, to mitigate the interference seen by each agile user at corresponding receiver subject to the constraint on SINR and the received power of each user. In other words our goal is to generalize the algorithm presented in [4-7, 10, 11] to a situation in which the channel between users and base station is non-ideal. Also we show that although some users are not able to change their waveforms, almost always, our algorithm converges to received power set and spreading code ensemble that along with fixed users' powers and waveforms well known as

Several works performed to overcome the effect of MAI in a multiple access system. These works fall under two major areas: optimization of received power, and signal design. Interference Avoidance (IA)

Generalized Welch Bound Equality (GWBE) sequences.

The paper organized as follows. We describe the system model and problem statement in Section II. In section III we formulate the problem as a non-cooperative game and verify the existence of Nash equilibrium for the game. Then we propose our waveform adaptation algorithm in section IV. Behaviour of waveform adaptation algorithm in network with mixtures of fixed and agile users, and numerical results obtained from simulations are presented in Sections 5 and 6, respectively. Finally we present the conclusions.

II. SYSTEM MODEL AND PROBLEM STATEMENT

A. System Model

We consider the uplink of a synchronous MC-CDMA system, where there are K simultaneous users that spread data on N orthogonal subcarriers using different spreading codes. The transmitted signal of the data symbol for k th user, $s_k(t)$ $k = 1, \dots, K$ in l th pulse duration is

$$s_k(t) = \sum_{i=1}^N \sqrt{p_k} b_k c_k^i e^{2\pi j(f_0 + f_d)t} p(t - lT), \quad (1)$$

where b_k and p_k are the symbol transmitted by k th user and its power, respectively. c_k^i for $i = 1, \dots, N$ is the i th chip of the spreading sequence corresponding to user k and f_d is the subcarrier separation. $p(t)$ in Equation (1) is a time shifted rectangular signaling pulse, given by

$$p(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq T \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

The received signal corresponding to k th user is

$$r_k(t) = \sum_{i=1}^N h_k^i \sqrt{p_k} b_k c_k^i e^{2\pi j(f_0 + f_d)t} p(t - lT), \quad (3)$$

where h_k^i is the frequency response of frequency selective channel to i th subcarrier of k th user and t_d is the time delay. The received signal to the base station can be expressed as:

$$r(t) = \sum_{k=1}^K r_k(t) + n(t) \quad (4)$$

where $n(t)$ is the additive Gaussian noise at the receiver. Let us represent the vector of MC-CDMA spreading code of user k as \mathbf{c}_k . Hence, Equation (4) can be written in matrix form as



$$\mathbf{r} = \sum_{k=1}^K \sqrt{p_k} b_k H_k \mathbf{c}_k + \mathbf{n}, \quad (5)$$

Since user's data are spreaded over multiple carriers, the channel gain matrix of k -th user is a diagonal matrix

$$H_k = \text{diag}(h_k^1, h_k^2, \dots, h_k^N)$$

and

$\mathbf{c}_k = [c_k^1, c_k^2, \dots, c_k^N]^T$ are the $N \times N$ channel matrix and spreading sequence vector of user k , respectively, and \mathbf{n} is an independent and identically distributed (i.i.d.) Gaussian vector with covariance \mathbf{W} , which is independent of the transmitted symbols. We note that all user spreading codes take values in the N -dimensional sphere with radius 1, i.e.

$$\mathbf{c}_k^T \mathbf{c}_k = 1 \quad \forall k = 1, \dots, K. \quad (6)$$

For the next analysis we define

$$\mathbf{c}_k^r = \frac{H_k \mathbf{c}_k}{\|H_k \mathbf{c}_k\|}, \quad (7)$$

and

$$p_k^r = p_k \|H_k \mathbf{c}_k\|^2, \quad (8)$$

as normalized received spreading sequence and received power, respectively.

B. Structure of Optimum Linear Receiver

It is well known that the MMSE receiver is the optimum linear receiver for multiuser detection, optimum in the sense of maximizing the SINR of each user. As mentioned in [3, Sec. III], MMSE receiver and matched filter (MF) show the same performance in presence of white noise at the optimal solution presented in [3] (i.e. optimum in the sense of maximizing the sum capacity or equivalently minimizing the TSC with minimum sum power that achieve users to their target SINRs). Thus, we assume that matched filters are employed at the receiver to detect user symbols. Since the receiver has no a priori knowledge about transmitted codewords, the orthogonal projection model is used to identify the users codeword at the receiver. Thus the use of normalized received codeword of users instead of transmitted codeword for matched filter is more relevant. In this case, the decision variable d_k for user k is:

$$\begin{aligned} d_k &= (\mathbf{c}_k^r)^T \mathbf{r} \\ &= (\mathbf{c}_k^r)^T \left(\sqrt{p_k^r} b_k \mathbf{c}_k^r + \sum_{\substack{i=1 \\ i \neq k}}^K \sqrt{p_i^r} b_i \mathbf{c}_i^r + \mathbf{n} \right). \end{aligned} \quad (9)$$

So the SINR for user k becomes:

$$\gamma_k^{MF} = \frac{p_k^r}{(\mathbf{c}_k^r)^T \mathbf{R}_k (\mathbf{c}_k^r)}, \quad (10)$$

where

$$\mathbf{R}_k = \mathbf{R} - p_k H_k \mathbf{c}_k \mathbf{c}_k^T H_k, \quad (11)$$

is the interference plus noise covariance matrix and

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^T] = \sum_{k=1}^K p_k H_k \mathbf{c}_k \mathbf{c}_k^T H_k + \mathbf{W}, \quad (12)$$

is the covariance matrix of received signal.

The interference function of user k can be defined as denominator of k th user's SINR

$$i_k = (\mathbf{c}_k^r)^T \mathbf{R}_k (\mathbf{c}_k^r). \quad (13)$$

Note that the interference function of user k is independent of power p_k .

C. Admissibility of users

In our setup individual users adjust their spreading sequence and powers to meet a set of specified target SINRs $\gamma^* = \{\gamma_1^*, \dots, \gamma_k^*, \dots, \gamma_K^*\}$. But for this reason, the admissibility condition must be satisfied. K users with the above target SINRs are admissible in CDMA systems with processing gain N , if there exist an allocation of signature sequences and powers such that the SINR of the users are at least equal to their target SINRs. The user capacity or admissibility region is defined to be the set of all admissible SINR requirements

Admissibility condition is presented for the case that no power constraint is considered [3].

K users with SINR requirements $\{\gamma_1^*, \dots, \gamma_k^*, \dots, \gamma_K^*\}$ are admissible in the system with processing gain N if and only if

$$\sum_{k=1}^K e(\gamma_k^*) < N \quad (14)$$

where $e(\gamma_k^*)$ is the effective bandwidth and defined as

$$e(\gamma_k^*) = \gamma_k^* / (\gamma_k^* + 1) \quad (15)$$

This is because the user capacity region can then be described as the region where the sum of the effective bandwidths of the users does not exceed the processing gain. The bound (14) known as user capacity or admissibility region [2, 3].

As mentioned in [2] this result assumes that there is no constraint on the received power and thus the scenario of colored additive noise has no effect on this user capacity region.

If we put received power constraint on individual users, constructing the capacity region becomes very complicated. In [3] the capacity region for special case in which the target SINR of users are the same, is computed. We can test the admissibility of users with given target SINRs and received power constraints by following theorem. Note that in this theorem white noise is considered.



Theorem 1: K users (each having target SINR γ_k^* , $k = 1, \dots, K$) are admissible in the system with processing gain N , received power constraint \bar{P} and noise covariance matrix $\sigma^2 \mathbf{I}$ if and only if the target SINRs pass the following test

- 1) Compute the parameter $e(\gamma_k^*)$ for all users.
- 2) Sort effective bandwidth of users from largest to smallest
- 3) Consider the j largest users as oversized if for users 1 to j we have

$$e(\gamma_i^*) > \frac{\sum_{k=1}^j e(\gamma_k^*)}{N - j}, i = 1, \dots, j \quad (16)$$

- 4) We must have

$$e(\gamma_i^*) < \frac{\bar{P}}{\bar{P} + \sigma^2}, i = 1, \dots, j \quad (17)$$

and

$$\sum_{k=j+1}^K e(\gamma_k^*) < (N - k) \left(1 - \frac{\sigma^2 e(\gamma_{j+1}^*)}{\bar{P}} \right) \quad (18)$$

Note that admissibility condition without power constraint is an especial case of Theorem 1 ($\bar{P} \rightarrow \infty$).

The admissibility region with equal received power constraint is the set of all SINR requirements that pass the above admissibility test.

III. FORMULATION AS NONCOOPERATIVE GAME

As mentioned in [6] the users of the system can be considered as player of a non-cooperative game. The game is non-cooperative in the sense that each user maximizes his utility without considering how the selected strategy affects other users' performance.

In this paper the transmitters adapt their received powers and codewords by adapting their transmitted waveforms to mitigate the interference, subject to constraints on users' SINRs and transmitted codeword norm. Thus we model our game based on received power and codeword. In our paper we consider the utility function of a given user to be the product between its received power and its corresponding interference function.

$$u_k = -p_k^r i_k \quad (19)$$

Note that our aim is to adapt the received power and codeword in order to maximize the performance of the network. Thus (19) cannot be written in the form

$$u_k = -p_k \mathbf{c}_k^T H_k \mathbf{R}_k H_k \mathbf{c}_k$$

and optimized in \mathbf{c}_k and p_k . The utility function (19) can be written in the following form

$$u_k(p_k^r, i_k(\mathbf{C}^r, \mathbf{p}_{-k}^r)), k = 1, \dots, K \quad (20)$$

where \mathbf{C}^r is the received codeword matrix whose columns are normalized received codewords, and \mathbf{p}_{-k}^r is the vector of received powers in that k th received

power excluded. Moreover the utility function is decreasing function of interference for constant received power p_k^r . We say that a utility function satisfying the above property is separable, in regard to the two parameters, received power and codeword. Similarly a game with such a utility function is named separable game [14]. Thus we can separate the joint power and sequence control game \mathbf{G} into two subgames. We analyze these two subgames separately in following subsections.

A. Codeword adaptation subgame

In this section we present the codeword adaptation subgame \mathbf{G}_c assuming that the received powers of users are constant. We find the best response of this subgame. For this subgame, each user selects his strategy to maximize their corresponding utility function for a given set of received powers, that is

$$\begin{aligned} \max_{\mathbf{c}_k} u_k \Big|_{\mathbf{p}^r = \text{cte}}, k = 1, \dots, K \\ \text{subject to } \mathbf{c}_k^T \mathbf{c}_k = 1 \end{aligned} \quad (21)$$

We can easily rewrite the utility function (19) as below

$$u_k = -p_k^r \frac{(\mathbf{c}_k^r)^T \mathbf{R}_k (\mathbf{c}_k^r)}{(\mathbf{c}_k^r)^T (\mathbf{c}_k^r)} \quad (22)$$

that is in form of scaled Rayleigh quotient. Note that this does not change the value of the utility function for a given unit norm received signature sequence. Recall from linear algebra [15, p.348] that Rayleigh quotient is minimized by the eigenvector corresponding to the minimum eigenvalue of the interference plus noise covariance matrix. This implies that the best strategy for agile user k is a greedy interference avoidance procedure [13] in which user k 's received spreading sequence \mathbf{c}_k^r is replaced by the minimum eigenvector of matrix \mathbf{R}_k which minimizes the effective interference corrupting user k 's signal at the receiver [16]. We show that this choice of received spreading sequence \mathbf{c}_k^r is equivalent to replace the spreading sequence of user k by the minimum eigenvector of matrix $H_k^{-1} \mathbf{R}_k H_k$. As mentioned before, the solution of problem (21) is the inferior eigenvector of matrix \mathbf{R}_k , i.e.

$$\mathbf{R}_k \mathbf{c}_k^r = \lambda_k^{\min} \mathbf{c}_k^r, \text{ for } k = 1, \dots, K, \quad (23)$$

where λ_k^{\min} is the minimum eigenvalue of matrix \mathbf{R}_k .

Using Equations (7) and (23) we have

$$\mathbf{R}_k \frac{H_k \mathbf{c}_k}{\|H_k \mathbf{c}_k\|} = \lambda_k^{\min} \frac{H_k \mathbf{c}_k}{\|H_k \mathbf{c}_k\|}, \quad (24)$$

and hence

$$(H_k^{-1} \mathbf{R}_k H_k) \mathbf{c}_k = \lambda_k^{\min} \mathbf{c}_k. \quad (25)$$

It is worth mentioning that, the equality $\text{eig}(AB) = \text{eig}(BA)$ implies that λ_k^{\min} is the minimum



eigenvalue of $H_k^{-1} \mathbf{R}_k H_k$, as well. Moreover, norm of $H_k \mathbf{c}_k / \|H_k \mathbf{c}_k\|$ is independent of $\|H_k \mathbf{c}_k\|$. Thus, (23), (24) and (25) are reversible.

In each spreading sequence update we have:

$$\mathbf{c}_k(t+1) = \mathbf{x}_k(t), \forall k = 1, \dots, K \quad (26)$$

where $\mathbf{x}_k(t)$ is the minimum eigenvector of $H_k^{-1} \mathbf{R}_k(t) H_k$ and t is the adaptation instance.

In [17] the interference avoidance problem for fixed power is modeled in a game theoretic framework and is formulated as a potential game. The existence of Nash equilibrium and convergence properties, for best and better response iterations (wherein users choose sequences that maximize or increase their utility, respectively) of potential games are then derived. It is shown that eigen-iterations algorithm (26) for interference avoidance converges to the global solution when noise is added [17]. This algorithm was previously shown to converge to the global optima only using class warfare techniques [18] which are not amenable to a distributed implementation.

The received codeword matrix \mathbf{C}^r is the Nash equilibrium of codeword adaptation subgame if for every $k = 1, \dots, K$

$$\begin{aligned} u_k(\mathbf{c}_1^r, \dots, \mathbf{c}_{k-1}^r, \mathbf{c}_k^r, \mathbf{c}_{k+1}^r, \dots, \mathbf{c}_K^r) &\geq \\ u_k(\mathbf{c}_1^r, \dots, \mathbf{c}_{k-1}^r, \mathbf{c}_k^r, \mathbf{c}_{k+1}^r, \dots, \mathbf{c}_K^r), &\forall \mathbf{c}_k^r \in \mathbf{C}_k \end{aligned} \quad (27)$$

Where \mathbf{C}_k is the strategy space of \mathbf{G}_c for user k . In other words \mathbf{C}_k is the set of all unit norm vectors in \mathbb{R}^N . Thus \mathbf{x}_k is the best strategy of sequence control subgame for user k .

B. Power adaptation subgame

In power control subgame \mathbf{G}_p , the codeword of all users assumed to be fixed. Individual users try to maximize their utility functions subject to the target SINR constraint met. i.e.

$$\begin{aligned} \max_{P_k} u_k |_{\mathbf{C}=\text{cte}}, k = 1, \dots, K \\ \text{subject to } SINR_k \geq \gamma_k^* \end{aligned} \quad (28)$$

Note that power adaptation subgame can be considered as a convex game according to definition stated in [19], since the user utility function is linear in P_k . Thus according to [19, Theorem 1] there exist an equilibrium point for power control subgame.

As in [20] we can change the inequality constraint in (28) to the equality constraint. Thus we can rewrite the problem (28) as follows

$$\begin{aligned} \max_{P_k} u_k |_{\mathbf{C}=\text{cte}}, k = 1, \dots, K \\ \text{subject to } SINR_k = \gamma_k^* \end{aligned} \quad (29)$$

The solution of (29) is straight forward and the best response strategy for received power adaptation subgame for user k is:

$$P_k^r = \gamma_k^* (\mathbf{c}_k^r)^T \mathbf{R}_k (\mathbf{c}_k^r) = \gamma_k^* i_k \quad (30)$$

Thus from (8) we have

$$P_k(t+1) = \min \left\{ P_{\text{sup}}, \frac{\gamma_k^* i_k(t)}{\|H_k \mathbf{c}_k(t)\|^2} \right\} \quad (31)$$

Where P_{sup} is maximum power that can be transmitted by user k .

C. Existence of Nash equilibrium for joint power and Codeword Adaptation Game

According to [14, Theorem 2] Nash equilibrium of the power and sequence control game exists if 3 properties are satisfied. Now we check these 3 properties as follows.

1) Property 1: the Game Is Separable

We checked this property in section III where we presented the utility function.

2) Property 2: the Power Is Continuous Function of Given Interference

[14, Property 2]: Given any interference i_k there is a power $P_k^{r*}(i_k)$ that maximizes the utility of user k , that is

$$u_k(P_k^{r*}(i_k), i_k) = \max_P u_k(P, i_k) \quad (32)$$

and the function $P_k^{r*}(i_k)$ is a continuous function of i_k .

It is easy to show that this property is satisfied. Given any interference i_k , the k -th user can adjust his power so that the utility function is maximized. From (31) the optimum power can be chosen as

$$P_k^{r*}(i_k) = \min \left\{ \|H_k \mathbf{c}_k(t)\|^2 P_{\text{sup}}, \gamma_k^* i_k \right\} \quad (33)$$

Clearly this power is a continuous function of i_k .

3) Property 3: There Exist a Nash Equilibrium for Sequence Control Game

[14, Property 3]: For any \mathbf{p}^r there exists an equilibrium point $\mathbf{C}^{r*}(\mathbf{p}^r)$ in the subgame $\mathbf{G}_c(\mathbf{P})$

such that the function $i_k(\mathbf{C}^{r*}(\mathbf{p}^r), \mathbf{p}_{-k}^r)$ is continuous as a function of \mathbf{p}^r , for $k = 1, \dots, K$.

As mentioned in Sec.I, according to [17, Sec.V], for a given \mathbf{p}^r , codeword adaptation subgame has a Nash equilibrium $\mathbf{C}^{r*}(\mathbf{p})$. It is shown in [14, Proposition 4] that the interference function $i_k(\mathbf{p}_{-k}^r, \mathbf{S}^{r*}(\mathbf{p}^r))$ is a continuous function of \mathbf{p}^r .

Thus properties 1-3 in [14] is satisfied and joint power and sequence control game has Nash equilibrium.

D. Controlled Better Response Strategy Instead of Best Response Strategy

As mentioned in [21], applying (26) may lead to new user spreading sequences that are distant from the



current user spreading sequence in signal space, and/or (31) may cause abrupt power variation. This behavior is not desirable since it may lead to increase the error probability at the receiver, especially when connection loss between the transmitter and the base station occurs. Thus, as in [6], the users change their spreading sequence and powers in small increments. We define the spreading sequence update rule as

$$\mathbf{c}_k(t+1) = \frac{\mathbf{c}_k(t) + m\beta\mathbf{x}_k(t)}{\|\mathbf{c}_k(t) + m\beta\mathbf{x}_k(t)\|}, \quad (34)$$

where

$$m = \text{sgn}\left(\mathbf{c}_k(t)^T H_k^T H_k \mathbf{x}_k(t)\right)$$

and β limits the Euclidean distance between two successive updates in signal space. Also we define the power update rule as

$$p_k(t+1) = (1-\mu)p_k(t) + \mu \frac{\gamma_k^* i_k(t)}{\|H_k \mathbf{c}_k(t)\|^2} \quad (35)$$

where $0 < \mu < 1$. The disadvantage of better response strategy is lower speed of convergence in compare with best response dynamic.

IV. DYNAMIC WAVEFORM ADAPTATION ALGORITHM

In this section we present our waveform adaptation algorithm considering non ideal channel between users and base station, and SINR. A formal statement of the algorithm is given below:

1) *Initial setup*: Start with randomly selected spreading sequences, powers, and channel gain matrices.

2) *Admissibility Check*: IF the target SINRs satisfy the admissibility condition (14) GO TO step 3 ELSE STOP: users are not admissible.

3) *Convergence Check*: IF the difference between two successive convergence factor (such as SINR) are greater than ε GO TO step 4, ELSE STOP: an optimal configuration has been reached.

4) *Adaptation Stage*: FOR each user $k = 1, \dots, K$ DO:

a) calculate current correlation matrix using Equation (11) and then $H_k^{-1} \mathbf{R}_k(t) H_k$.

b) Determine the minimum eigenvector $\mathbf{x}_k(t)$.

c) Replace the current spreading sequence using update Equation (34).

d) Replace the current power using update Equation(35).

5) GO TO step 3.

Extensive simulations showed that this algorithm converges to optimal fixed point stated in [3] for white noise and to optimal fixed point stated in [18] for colored noise. In next section we study the behaviour of waveform adaptation algorithm in a network with mixture of agile and fixed users.

V. BEHAVIOUR OF WAVEFORM ADAPTATION ALGORITHM IN NETWORK WITH MIXTURES OF FIXED AND AGILE USERS

In our scenario we consider a mixture of fixed and agile users. The fixed users are able only to adjust their powers to meet the SINR constraint, as in current wireless networks. With no loss of generality we assume that the agile (unlicensed) users is numbered from 1 to $K_a < K$. Thus,

$$\mathbf{R} = \underbrace{\sum_{i=1}^{K_a} p_i^r (\mathbf{c}_i^r) (\mathbf{c}_i^r)^T}_{\mathbf{R}_a} + \underbrace{\sum_{i=K_a+1}^K p_i^r (\mathbf{c}_i^r) (\mathbf{c}_i^r)^T}_{\mathbf{R}_f} + \underbrace{\sigma^2 \mathbf{I}_N}_{\mathbf{W}} \quad (36)$$

where \mathbf{R}_a and \mathbf{R}_f are the covariance matrices of agile and fixed users received signals, respectively.

Note that from agile users' point of view, the fixed user signals can be viewed as colored noise. It is shown that if there are $K_a < N$ agile users, then the interference experienced by at least one agile user can be reduced (while not increasing the interference seen by other agile users) unless the set of agile user codewords is contained in the space spanned by the K_a eigenvectors of \mathbf{R}_f with the smallest eigenvalues [13, Theorem 3]. Thus to achieve minimum interference, the $\{\mathbf{c}_k^r, k = 1, \dots, K_a\}$ must reside in the space spanned by the $Q \leq K_a$ eigenvectors of \mathbf{R}_f with smallest eigenvalues. This feature is of benefit, on average, to the fixed users as well.

In this work we focus on situation in which the number of agile users is greater than processing gain of CDMA system. Note that in this scenario the unlicensed and licensed users use the same spectrum.

A. Formulation at the fixed point

Codeword adaptation rule in (34) implies that, in fixed point of waveform adaptation algorithm, the received codeword of each user is minimum eigenvector of interference plus noise covariance matrix. As shown in (23)

$$\mathbf{R}_k \mathbf{c}_k^r = \lambda_k^{\min} \mathbf{c}_k^r, \quad \forall k = 1, \dots, K_a, \quad (37)$$

(23) followed by (11) implies that \mathbf{c}_k^r is the eigenvector of received signal covariance matrix as well. i.e.

$$\mathbf{R} \mathbf{c}_k^r = (\lambda_k^{\min} + p_k) \mathbf{c}_k^r = \lambda_k \mathbf{c}_k^r \quad (38)$$

Furthermore (30) implies that the received power of user is

$$p_k^r = \gamma_k^* \lambda_k^{\min} = e(\gamma_k^*) \lambda_k \quad (39)$$

B. Properties of Codeword Ensembles at fixed point

We can show that in fixed point of the algorithm the, the covariance matrices of agile and fixed users commute¹ and this implies that \mathbf{R}_a and \mathbf{R}_f are jointly

¹Two matrices A and B which satisfy $AB = BA$ are said to be commuting under matrix multiplication.



diagonalizable. In other words at the fixed point of algorithm, the agile and fixed users share the same basis functions.

As stated in [18] at optimal fixed point of interference avoidance algorithm with fixed power and colored noise, the power distribution of users is classes of water fillings. Thus, a fixed-pointensemble in general has a received codeword power distribution as depicted inFig. 1.

In Fig. 1 $\lambda_I, \lambda_{II}, \lambda_{III}$ are the distinct eigenvalues of

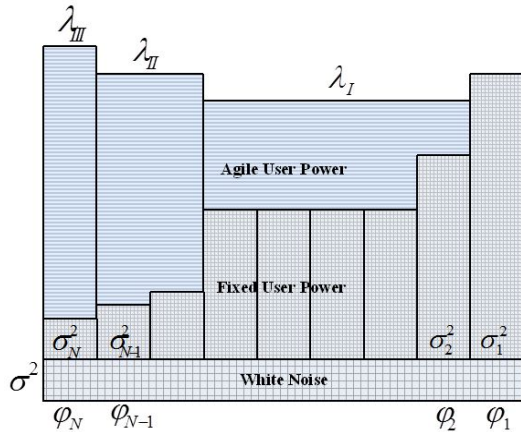


Fig. 1 Illustration of power distribution of optimal fixed point.

matrix \mathbf{R} corresponding to user codewords, $\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2$ are the eigenvalues of fixed users covariance matrix \mathbf{R}_f and $\phi_1, \phi_2, \dots, \phi_N$ are the basis vectors which diagonalize \mathbf{R}_a and \mathbf{R}_f . It is easy to show that if users 1 to k has codewords corresponding to λ_I , then the relation between powers and corresponding eigenvalues are as below

$$\sum_{i=1}^k \lambda_i e(\gamma_i^*) = \sum_{j=1}^k (\lambda_i - \sigma_j^2 - \sigma^2)^+ \quad (40)$$

Where k is the maximum index of basis vectors corresponding to eigenvalue λ_I . This relation is true for users with other corresponding eigenvalues as well.

We can name the users with codeword corresponding to eigenvalues $\lambda_{II}, \lambda_{III}$ as oversized agile users in the presence of fixed users.

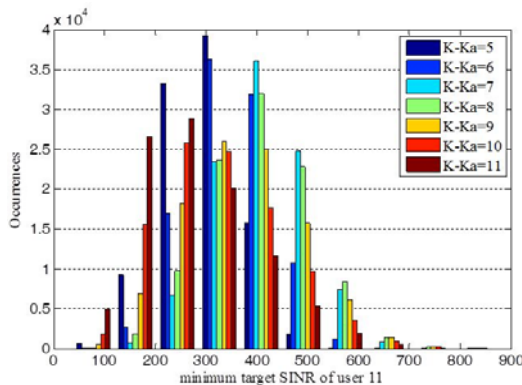


Fig. 2 Monte Carlo simulation to investigate occurrences of oversized agile user in presence of fixed users with randomly chosen codewords and powers in each run. $N=8, K=16, \dots, 25$ and $K_a=11$.

C. Effect of Waveform Adaptation Algorithm on Fixed Users

In this section, we present the properties of optimal power and spreading code ensemble obtained using algorithm stated in the previous section and analyse the effect on fixed users performance. We will show that almost always, although the fixed users unable to change their codewords, their codewords are the inferior eigenvector of corresponding interference plus noise covariance matrix.

Extensive simulations showed that for normal choice of agile user target SINRs, almost never a user is oversized. We show this by considering following trial.

Suppose that there are $K_a=11$ agile users with SINR requirements of

$$\gamma^* = \{1, 1.2, 1.4, 1.6, 1.8, 2, 2.5, 3, 3.2, 3.5, x\}$$

and K_f fixed users in $N=8$ signal dimensions. We compute the SINR requirement of agile user 11, “x” to be oversized. In this way we repeat the trial 100,000 times for various number of fixed users ($K_f = 5, \dots, 14$) and randomly generated fixed user codewords and powers. The results are depicted inFig. 2. We can see that the user 11 is oversized when it’s target SINR is very large compared with other target SINRs.

Thus we can restrict ourselves to situation in which no agile user is oversized. In this case agile users have a power and codeword ensemble that satisfies a simultaneous water filling like solution. The following theorem shows the effect of algorithm to fixed user performance.

Theorem 2: For $K_a > N$ at the fixed point of algorithm, the codeword of fixed users are the inferior eigenvector of their corresponding interference plus noise covariance matrix, if and only if the covariance matrix of agile users is invertible.

Proof: we define the $N \times K$ received spreading sequence matrix $\mathbf{C}^r = [\mathbf{c}_1^r, \dots, \mathbf{c}_K^r]$ and received power vector $\mathbf{P}^r = [p_1^r, \dots, p_K^r]$, where \mathbf{c}_k^r and p_k^r are received spreading sequence and powers of user k obtained from (7) and (8), respectively.

We can rewrite Equation (38) as follows

$$\mathbf{R} \mathbf{c}_k^r = (\lambda_k^{\min} + p_k^r) \mathbf{c}_k^r = \lambda \mathbf{c}_k^r, \quad \forall k = 1, \dots, K_a, \quad (41)$$

Where λ is the eigenvalue of received signal covariance matrix \mathbf{R} corresponding to received codeword of user k . From Equation (41) we have

$$(\mathbf{R}_i + p_i^r (\mathbf{c}_i^r) (\mathbf{c}_i^r)^T) \mathbf{c}_k^r = \lambda \mathbf{c}_k^r; \quad (42)$$

$$\forall k = 1, \dots, K_a \text{ and } i = K_a + 1, \dots, K,$$

Multiplying Equation (42) by $(\mathbf{c}_i^r)^T$ from left and $p_k^r (\mathbf{c}_k^r)^T$ from right yields



$$\begin{aligned}
 & (\mathbf{c}_i^r)^T \left(\mathbf{R}_i + p_i^r (\mathbf{c}_i^r)(\mathbf{c}_i^r)^T \right) p_k^r (\mathbf{c}_k^r)(\mathbf{c}_k^r)^T \\
 &= (\mathbf{c}_i^r)^T \lambda p_k^r (\mathbf{c}_k^r)(\mathbf{c}_k^r)^T, \quad (43) \\
 & \forall k = 1, \dots, K_a \text{ and } i = K_a + 1, \dots, K,
 \end{aligned}$$

After some mathematical manipulations we have

$$\begin{aligned}
 & (\mathbf{c}_i^r)^T \mathbf{R}_i \sum_{k=1}^{K_a} p_k^r (\mathbf{c}_k^r)(\mathbf{c}_k^r)^T \\
 &= (\mathbf{c}_i^r)^T \left(\lambda - p_i^r \right) \sum_{k=1}^{K_a} p_k^r (\mathbf{c}_k^r)(\mathbf{c}_k^r)^T
 \end{aligned} \quad (44)$$

for $i = K_a + 1, \dots, K$, or equivalently

$$(\mathbf{c}_i^r)^T \mathbf{R}_i \mathbf{R}_a = (\mathbf{c}_i^r)^T \lambda_i^{\min} \mathbf{R}_a; \forall i = K_a + 1, \dots, K. \quad (45)$$

For an invertible \mathbf{R}_a , Equation (45) can be reduced to

$$\mathbf{R}_i \mathbf{c}_i^r = \lambda_i^{\min} \mathbf{c}_i^r, \forall i = K_a + 1, \dots, K. \quad (46)$$

As a consequence, Equations (37) and (46) imply that all user spreading codes are the eigenvectors corresponding to the minimum eigenvalue of their interference plus noise covariance matrix. The inverse proof is straightforward. It is obvious from Fig. 1 that \mathbf{R}_a is invertible if the following inequality is satisfied.

$$\lambda - (\sigma_i^2 + \sigma^2) > 0, i = 1, \dots, N \quad (47)$$

where σ_i^2 is the i th eigenvalue of \mathbf{R}_f and λ is satisfy the following equality

$$\sum_{i=1}^N \left(1 - \frac{\sigma_i^2 + \sigma^2}{\lambda} \right)^+ = \sum_{i=1}^{K_a} e(\gamma_i^*) \quad (48)$$

An important property of GWBE sequences is that, each sequence is the minimum eigenvector of its corresponding \mathbf{R}_k [22]. This means that, minimum interference plus noise energy is experienced by each user.

Note that in this analysis we considered the general form in which the fixed users assign their codewords arbitrarily. Where fixed users assign their codewords orthogonally, the result is more straightforward. In this case, when (47) is not satisfied for some i , the fixed user corresponding to eigenvalue σ_i^2 will be orthogonal to all the agile users and other fixed users. In other words this user is oversized. Thus we can consider the SINR of fixed users in admissibility condition (14). Furthermore we can use the admissibility test in theorem 1 for agile and fixed users if received power constraint is considered. Another property of optimal Nash equilibrium is that, the matrix \mathbf{R} is scaled identity matrix.

VI. SIMULATION RESULTS

In this section, we provide numerical examples to verify our theoretical findings and algorithm.

First, we consider processing gain $N = 8$, $K_f = 5$ unconstrained fixed users and $K_a = 8$ agile users with the following target SINRs

$$\gamma^* = \{1, 1, 1.5, 1.5, 2, 2, 2.5, 2.5, 3\}$$

Which satisfy admissibility condition stated in (14). Background noise is also white with covariance matrix $\mathbf{W} = 0.1 \mathbf{I}_{8 \times 8}$ and the simulation constants are $\beta = \mu = 0.1$ and tolerance is $\varepsilon = 10^{-5}$. We run the algorithm with randomly chosen initial sequences and powers for agile users and following Walsh-Hadamard orthonormal sequences and arbitrary powers for fixed users. The channel matrices are chosen arbitrarily as well.

$$\mathbf{F} = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\mathbf{P} = [2.9202, 1.9519, 1.3692, 2.1239, 0.5135]$$

After the waveform adaptation algorithm performed, we obtain received codeword matrix \mathbf{C}_a^r and received power vector \mathbf{P}_a^r . The eigenvector and eigenvalue matrices of the covariance matrices corresponding to the agile and fixed users is illustrated at the bottom of the next page. It can be seen that, two covariance matrices use the same basis vectors and that the sum of eigenvalues corresponding to identical eigenvectors is the constant 4.13 and $\lambda = 4.23$. This verifies the water filling property shown in Fig. 1 and equations (47) and (48). Note that the last 3 eigenvectors in \mathbf{U}_a is the linear combination of the first 3 eigenvectors in \mathbf{U}_f . This can be shown by projection of last 3 eigenvectors in \mathbf{U}_a onto the eigenvectors in \mathbf{U}_f . For instance, we do this for last eigenvector in \mathbf{U}_a (say $\mathbf{u}_a^{(5)}$).

$$\mathbf{u}_a^{(5)} \mathbf{U}_f = [0.9042 \quad 0.0184 \quad -0.4266 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

Note that this vector has unit norm. Thus the space spanned by the last 3 eigenvectors in \mathbf{U}_a also spanned by first 3 eigenvectors in \mathbf{U}_f . Furthermore, one can show that the Walsh-Hadamard codeword assigned to each fixed user is the inferior eigenvector of its corresponding interference plus noise covariance matrix. The covariance matrix of received signal is

$$\mathbf{R} = 4.23 \mathbf{I}_8$$

In the second example, we consider the signal space dimension $N = 4$ and white noise with covariance matrix $\mathbf{W} = 0.1 \mathbf{I}_4$. The simulation constants are $\beta = \mu = 0.1$ and tolerance is $\varepsilon = 10^{-5}$. Also the number of users and target SINRs are supposed to be $K = 7$ (with $K_a = 5$ agile users) and



$\gamma^* = \{1,1.1,1.2,1.3,1.4,1.5,1.6\}$, respectively. Finally we consider the received power constraint, $\bar{P} = 4$. In our simulations we assume that, the fixed users adjust their powers to meet their SINR constraints. Note that the sum of effective bandwidths based on (15) is equal to $3.9332 < 3.9384$ and Equation (18) is satisfied. We begin our simulation with randomly generated spreading codes, powers and following channel matrices.

$$\begin{aligned}
 H_1 &= \text{diag} \{0.7744, 0.5968, 0.8818, 0.9997\} \\
 H_2 &= \text{diag} \{0.9886, 0.4762, 0.5393, 0.4142\} \\
 H_3 &= \text{diag} \{0.7645, 0.4665, 0.6445, 0.9304\} \\
 H_4 &= \text{diag} \{0.7289, 0.6214, 0.5250, 0.6646\} \\
 H_5 &= \text{diag} \{0.9737, 0.4744, 0.6825, 0.9141\} \\
 H_6 &= \text{diag} \{0.4260, 0.8150, 0.9874, 0.5700\} \\
 H_7 &= \text{diag} \{0.4803, 0.8112, 0.9457, 0.7665\}
 \end{aligned}$$

We define the $N \times K_a$ matrix C_a in which the columns are the transmitted spreading sequences by agile users. The algorithm yields the matrix C_a and transmitted power vector P as follows.

$$C = \begin{bmatrix} 0.7906 & 0.1046 & 0.4709 & 0.1083 & -0.0934 \\ -0.2789 & 0.2953 & -0.5694 & -0.7993 & -0.9818 \\ -0.1667 & -0.8205 & 0.0874 & 0.2560 & -0.1391 \\ -0.5191 & -0.4783 & 0.6681 & -0.5328 & 0.0899 \end{bmatrix}$$

$$P = [4.9547 \quad 3.3874 \quad 5.3339 \quad 4.8884 \quad 6.1251]$$

The received signal correlation matrix is:

$$R = 5.9878 I_4,$$

$$U_a = \frac{1}{\sqrt{8}} \begin{bmatrix} -1 & -1 & 1 & -1 & 1 & 0.6564 & -1.3980 & 0.7841 \\ -1 & -1 & 1 & -1 & -1 & -0.6564 & 1.3980 & -0.7841 \\ -1 & 1 & 1 & 1 & -1 & 1.7280 & 0.1141 & 0.0322 \\ -1 & 1 & 1 & 1 & 1 & -1.7280 & -0.1141 & -0.0322 \\ -1 & 1 & -1 & -1 & 1 & 0.4686 & 1.4229 & 0.8693 \\ -1 & 1 & -1 & -1 & -1 & -0.4686 & -1.4229 & -0.8693 \\ -1 & -1 & -1 & 1 & -1 & -0.6031 & -0.0891 & 1.6212 \\ -1 & -1 & -1 & 1 & 1 & 0.6031 & 0.0891 & -1.6212 \end{bmatrix}$$

$$D_a = \text{diag} \{1.2098 \quad 2.0061 \quad 2.1781 \quad 2.7608 \quad 3.6165 \quad 4.1300 \quad 4.1300 \quad 4.1300\}$$

$$U_f = \frac{1}{\sqrt{8}} \begin{bmatrix} 0.0634 & -0.2409 & -1.7141 & 1 & -1 & 1 & -1 & -1 \\ -0.0634 & 0.2409 & 1.7141 & -1 & -1 & 1 & -1 & -1 \\ -0.3750 & 1.4864 & -0.8061 & -1 & 1 & 1 & 1 & -1 \\ 0.3750 & -1.4864 & 0.8061 & 1 & 1 & 1 & 1 & -1 \\ 1.1518 & 1.2106 & 0.4559 & 1 & -1 & -1 & 1 & -1 \\ -1.1518 & -1.2106 & -0.4559 & -1 & -1 & -1 & 1 & -1 \\ 1.5902 & -0.5167 & -0.4520 & -1 & 1 & -1 & -1 & -1 \\ -1.5902 & 0.5167 & 0.4520 & 1 & 1 & -1 & -1 & -1 \end{bmatrix}$$

$$D_f = \text{diag} \{0 \quad 0 \quad 0 \quad 0.5135 \quad 1.3692 \quad 1.9519 \quad 2.1239 \quad 2.9202\}$$

which is within $O(10^{-4})$ tolerance from the corresponding matrix implied by [3]. This corresponds to a GWBE set.

We complete our work by illustrating the tracking ability of the algorithm for slowly changing channel. We assume that a steady state configuration is reached. Then we change the channel gains randomly by $(0 \sim \pm 10)\%$ of initial values. Fig. 3 shows that user SINRs, transmitted power and received powers vary smoothly after this channel variation.

We have a discontinuity at the instance of channel variation. The channel variation results in variation of $\|H_k c_k\|$ in equations (7), (8) and (10). Consequently, we have abrupt changes in users' SINRs and received power at the variation instance. Since the SINR of users is no longer equal to target SINRs, the algorithm continues to adapt, yielding the new powers and codeword vectors. Because the algorithm uses the better response strategy, transmitted powers change to their new values smoothly.

Note that convergence speed after channel variation is more than convergence speed with randomly chosen channel.

VII. CONCLUSION

In this paper, the problem of adaptive and distributed joint power and spreading sequence control for uplink of synchronous single cell MC-CDMA system is modeled and analyzed by a game-theoretic approach. The best strategy of two subgames of this separable game is derived. It is shown that the proposed game has at least one Nash equilibrium point. Distributed algorithm is presented based on greedy interference avoidance.



The behavior of the presented waveform adaptation algorithm in the presence of some users with fixed waveforms is analyzed and it is shown that interference plus noise corresponding to each fixed user has minimum energy in the direction of its signal.

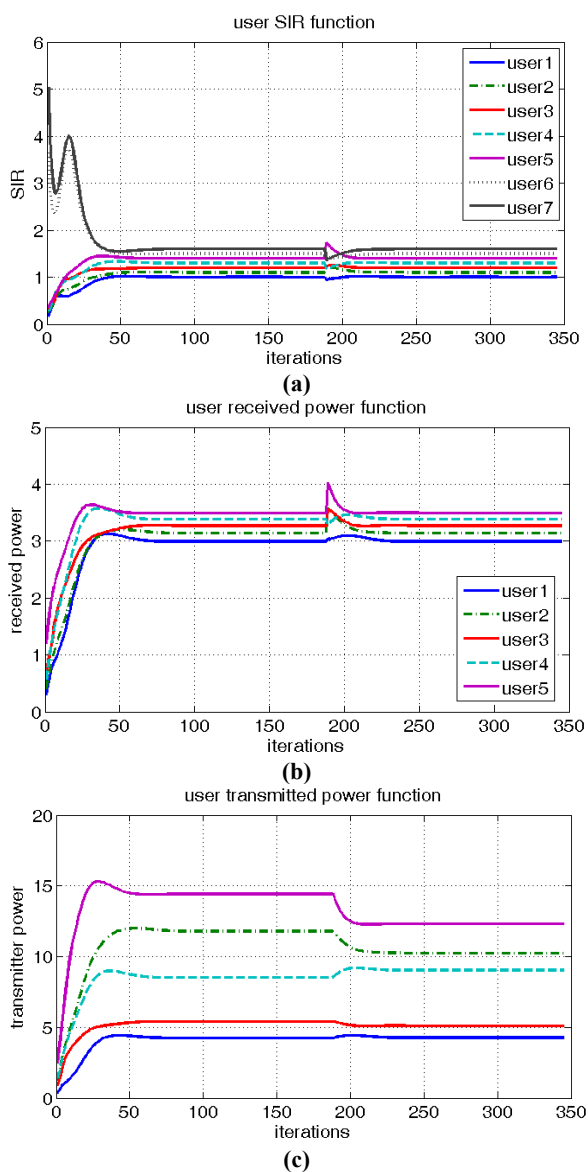


Fig. 3 Channel variation tracking. (a) SINR variation. (b) Received power variation. (c) Transmitted power

almost always, the waveform adaptation algorithm converges to a Nash equilibrium in which the

Finally, our theoretical results confirmed by the numerical examples. We have shown by simulation the ability of the presented algorithm to track the small changes in channel gains.

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