

Analysis Method for Spherical Dipole Antenna Array

Ahmad Hosseinbeig

Faculty of Electrical and Computer Engineering,
K. N. Toosi University of Technology
Tehran, Iran
Hosseinbeig@ieee.org

Manoochehr Kamyab

Faculty of Electrical and Computer Engineering,
K. N. Toosi University of Technology
Tehran, Iran
Kamyab@eetd.kntu.ac.ir

Javad S. Meiguni

Faculty of Electrical and Computer Engineering,
K. N. Toosi University of Technology
Tehran, Iran
Javad.meiguni@ieee.org

Received: September 4, 2012- Accepted: February 14, 2013

Abstract— Rigorous mathematical Method of Moments (MoM) for analyzing various radiating spherical structures is presented in this paper by using Dyadic Green's Functions (DGF) in conjunction with Mixed Potential Integral Equation (MPIE) formulation. With the aid of linear Rao-Wilton-Glisson (RWG) triangular basis functions and by converting spherical DGF to Cartesian DGF, a conformal dipole antenna over a Perfect Electric Conductor (PEC) sphere is analyzed. Mutual couplings between elements of a conformal dipole antenna array in an unbounded free space and over a conducting sphere are also investigated. Good agreement between the results obtained from the proposed method and asymptotic approximation as well as those of commercial simulator packages shows accuracy and high convergence speed of the presented method.

Keywords- DGFs transformation, dyadic Green's function, method of moment, spherical antennas.

I. INTRODUCTION

By using DGF, a field component in an arbitrary direction can be expressed in terms of a current vector component [1],[2]. Electromagnetic fields in a specific direction have been calculated for a small electric and magnetic dipole located inside a multilayer sphere [3]. In [4], an antenna structure over cylindrical shell has been analyzed by using spectral domain Green's functions. The resonance problem of a circular microstrip disk mounted on a spherical surface has been studied theoretically by utilizing Green's function formulation in spectral domain [5]. Closed form

equations using DGF containing a series of spherical harmonics have been used to evaluate electric and magnetic fields in such structures [2]. Archimedean spiral antennas printed on a multilayer dielectric sphere have been investigated in [6] utilizing the MoM. In [7], full-wave analysis of an arbitrary shape antenna placed on a layered sphere has been studied using DGF in combination with MPIE formulation for input impedance problems. In [8], dyadic Green's function has been expanded asymptotically which results in a significant increase in convergence rate of spherical harmonics series. With this approach, conformal antenna problems have been solved by evaluating fundamental coefficients in [9] using

spherical Bessel and Hankel functions approximation for large arguments. This technique is efficient for computation of input impedance for conformal antennas mounted on spherical layers. It is to be noted that due to different radial distances of field and source points, asymptotic approach cannot be used in computation of near or far field components of electromagnetic fields.

In this paper, explicit formulas are extracted to solve various radiating structures where source region is divided into linear triangles. Then a spherically conformal dipole antenna fed at its center located over a PEC sphere is analyzed. Next, the input admittance and radiation pattern of such a dipole antenna are calculated by using DGF. To increase spherical harmonics series convergence speed, the double summation in the series is transformed into a single summation by using associated Legendre function theorem. DGF transformation from spherical to Cartesian coordinates is also accomplished in this paper, which is used to efficiently compute radiation fields of various spherical antenna structures with linear triangular mesh generations. Mutual couplings between the elements of a conformal dipole antenna array located in free space or over a conducting sphere core are studied. Comparison of the results obtained from the proposed method with those of CAD simulations clearly shows the ability and accuracy of presented method.

II. THEORY

Figure 1 illustrates a conformal dipole antenna structure located in the vicinity of a PEC sphere. As shown in this figure, the structure can be divided into three regions. The conformal antenna is located at the boundary of layers 1 and 2. Region 3 is considered as a conducting sphere. In this case, the PEC is modeled by $\epsilon \rightarrow \infty, \mu \rightarrow 0$ [7]. Therefore, propagation constant in layer 3 is finite and numerical modeling of the antenna is feasible [7].

A. Method of Moment Formulation

One of the efficient numerical methods with high preprocessing gain for analyzing electromagnetic structures is the method of moment where the source region must be divided into cells. In this method, the unknown functions which are usually the source currents or charges are obtained via an integral equation formulation with appropriate Green's function. Such integral equations can be in space domain, spectral domain, or both of these two domains. In general due to meshing the finite area source, methods based on integral equation formulations are more accurate and require less memory and time. Dividing the source region into small triangles and considering the common edge between two cells as a current element, and expanding the current into triangular basis functions (\mathbf{f}_n), the source current can be defined as [10]:

$$\mathbf{J}_s = \sum_{n=1}^N I_n \mathbf{f}_n, \tag{1}$$

where N is the number of non-boundary edges and I_n are unknown coefficients which should be obtained

from MPIE formulation [10]. Therefore, if the source region is segmented by linear triangular cells, unknown current coefficients in the antenna can be determined by applying surface RWG basis functions and satisfying the boundary conditions [10]. Since tangential component of electric field vanishes on perfect conducting metal, impedance matrix for MPIE equation can be written as [7]:

$$Z_{pq} = - \iint_{s'} \iint_{s''} \left[j\omega f_p(\mathbf{r}') \overline{\mathbf{G}}_A f_q(\mathbf{r}'') + (\nabla f_p(\mathbf{r}') G_\psi(\nabla f_q(\mathbf{r}'')) \right] ds' ds'', \tag{2}$$

in which $\overline{\mathbf{G}}_A$ and G_ψ are the magnetic dyadic and the electric scalar Green's functions, respectively. Galerkin's method is applied for test functions and electric field is considered as:

$$\mathbf{E} = -j\omega \mathbf{A} - \nabla \psi \tag{3}$$

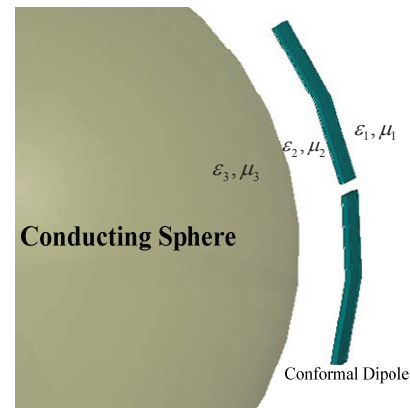


Figure 1. A conformal dipole antenna over a PEC sphere

Integration over the testing triangles can be avoided by using the centers of field cells and approximate Galerkin's method. Applying 3-point Gauss quadrature is sufficient for integration over source triangles [7].

B. Antenna in the Vicinity of Conductor Sphere

For full wave analysis of conformal dipole antenna over PEC sphere, the scattering components of electric field are important. These components can be obtained by using current distribution and field points in layer 1 which leads to [9]:

$$\overline{\mathbf{G}}_{es}^{(11)} = \frac{jk_1}{4\pi} \sum_{n=0}^{\infty} \sum_{m=0}^n \left[\begin{matrix} (2-\delta_m^0) \frac{2n+1}{n(n+1)} \frac{(n-m)!}{(n+m)!} \\ \left\{ \begin{matrix} b_M^{11} \mathbf{M}_{mn}^{(2)}(k_1) \mathbf{M}_{mn}^{\prime(2)}(k_1) \\ + b_N^{11} \mathbf{N}_{mn}^{(2)}(k_1) \mathbf{N}_{mn}^{\prime(2)}(k_1) \end{matrix} \right\} \end{matrix} \right], \tag{4a}$$

where δ_m^0 is the Kronecker delta and \mathbf{M}, \mathbf{N} vectors are Eigen vectors in spherical coordinates system with orthogonal properties explained in various references. Superscript (2) denotes spherical Hankel Function of the Second Kind. $b_{M,N}^{11}$ are the dyadic coefficients expressed as [9]:

$$b_{M,N}^{11} = - \frac{R_{F2}^{H Y} T_{F1}^{H Y} + R_{F1}^{H Y} T_{P1}^{H Y}}{R_{F2}^{H Y} R_{P1}^{H Y} T_{F1}^{H Y} + T_{P1}^{H Y}} \tag{4b}$$



where $R_{F1}^{H,V}$, $R_{F2}^{H,V}$, $T_{F1}^{H,V}$, $T_{P1}^{H,V}$ and $R_{P1}^{H,V}$ are as presented in [9].

The convergence speed of series depends on dielectric constants and radius of spherical layers.

Calculation of the double-summation of above DGF is exhausting and time-consuming since there are spherical harmonics in wave equation solution in spherical coordinates. The double summation in the spherical DGF can be reduced to an expression with only single summation by using the following relation [11]:

$$P_n(\cos \gamma) = \sum_{m=0}^n \left[\frac{(2 - \delta_m^0)(n-m)!}{(n+m)!} P_n^m(\cos \theta) \times P_n^m(\cos \theta') \cos(m(\varphi - \varphi')) \right], \quad (5)$$

where $\cos \gamma = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\varphi - \varphi')$.

C. Spherical to Cartesian Transformation of DGF

In order to obtain electromagnetic field components, we require multiplying dyadic Green's function and current element vector components for the case of interest. It is crucial to note that such multiplication must be performed in the same coordinates.

For the case that conformal antenna area is divided in curvilinear triangles (as shown in Fig.2 (a)), current components to be considered are J_φ and J_θ which are in harmony with spherical components of DGF [7]. However choosing curvilinear meshing in general can be complicated as compared with linear triangular meshing considered in this paper.

As current elements on common edges of linear meshes have Cartesian components, a new approach for dyad and vector multiplication is presented in this paper. Converting a current vector \mathbf{V} which connects two vertices of a triangle, from Cartesian to spherical coordinates results to non-unique vectors because J_φ and J_θ are different in each point of the edge such as \mathbf{V}_x shown in Fig. 2(b).

The drawback stated above can be greatly reduced if we employ the centers of the field and source triangles in DGF calculation in spherical coordinates. Thus there are unique transformations of vectors from spherical to Cartesian and therefore conversion of a spherical dyad to a Cartesian dyad can be exactly implemented. Then the electric field vector can be expressed as:

$$\mathbf{E}^{(Cartesian)} = -j \omega \mu_f \iint_s \overline{\mathbf{G}}^{(Cartesian)} \cdot \mathbf{J}^{(Cartesian)}(\mathbf{r}') ds' \quad (6)$$

Thus by using triangular linear meshes in Cartesian coordinates only $\overline{\mathbf{G}}$ needs to be converted from spherical to Cartesian coordinates. For this purpose, each unit vector should be transformed from spherical to Cartesian coordinates. As DGFs represent interactions between field and source points, the first and second vectors of each dyad correspond to field and source points respectively.

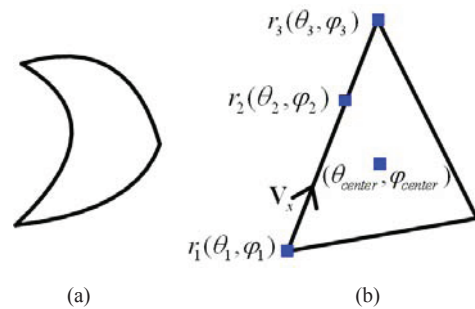


Figure 2. (a) Curvilinear triangle, (b) Linear triangle in the Cartesian coordinates with unique center point

Based on the argument stated above, all nine components of a spherical dyad can be converted to a Cartesian dyad. As an example the conversion equation of $\hat{r} \hat{r}$ component of a dyad is extracted as follows:

$$\hat{r}\hat{r} = (\sin \theta_f \cos \varphi_f \hat{x} + \sin \theta_f \sin \varphi_f \hat{y} + \cos \theta_f \hat{z}) (\sin \theta_s \cos \varphi_s \hat{x} + \sin \theta_s \sin \varphi_s \hat{y} + \cos \theta_s \hat{z}), \quad (7)$$

in which subscripts f and s refer to field and source points respectively. Accordingly each spherical dyadic function of form $\overline{\mathbf{G}}_1^{(fs)}$ can be converted to a Cartesian dyadic function $\overline{\mathbf{G}}_2^{(fs)}$.

It should be noted that input impedance formula of a conformal antenna over a spherical shell can be extracted using addition theorem for spherical Hankel functions [11]. For conformal antenna over a multilayer sphere, when both source and field points are at the same distance from the sphere center, we do not need to consider $\overline{\mathbf{G}}_A^{(fs)}$ and it is only enough to compute $G_{\psi}^{(fs)}$. Thus input impedance of antennas located on a multilayer sphere can be obtained using DGF or asymptotic approximation formulas. Asymptotic approximation method yields a higher convergence speed in calculation antenna input impedance but it cannot be utilized for radiation pattern determination since field and source points are not at the same distance from the sphere center in this case.

D. Array of Conformal Dipole Antennas

The presented method can be utilized to compute mutual couplings between antenna elements of a conformal dipole antenna array located in free space or above a PEC sphere. Scattering matrix can be computed as follows [12]:

$$[S] = (Y_0 [I] + [Y])^{-1} (Y_0 [I] - [Y]), \quad (8)$$

where I is the identity matrix and Y is admittance matrix. Y_0 is the characteristic admittance of feed network. As delta gap voltage is used to excite the electric dipoles, it is advantageous to compute scattering matrix from admittance matrix. The components of admittance matrix are defined as [12]:

$$Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_l=0, l \neq j} \quad (9)$$



Therefore, for an array consisting of N elements, coupled currents due to an excited element on the center of the unexcited elements should be calculated.

III. RESULTS

To validate the present computation, a spherically conformal dipole antenna which is located at a 5.32-cm distance from the origin is considered. The antenna physical length is $\lambda/2$ with λ being the wavelength at antenna center frequency ($f_0 = 3$ GHz). The antenna is located 0.32cm above a 5cm radius sphere and the medium between the antenna and the sphere is assumed to be free space. The antenna input admittance and radiation pattern are determined and the results are compared with commercial software results. It is expected that the antenna response is similar to a linear thin $\lambda/2$ dipole antenna which is presented in [13]. In this example, the antenna has negligible thickness in comparison with wavelength and is located in xy as shown in Fig.1 (b). The antenna is divided to 120 triangular linear meshes and there are 119 common edges between two adjacent plus and minus triangles. The delta gap voltage source is used to excite the dipoles. Impedance matrix and current distribution on common edges are computed using MoM. Besides utilizing DGF, asymptotic approximation method is also used which shows good agreement with the proposed method.

In this case, at least 60 terms of electric scalar Green’s function should be considered to obtain exact solutions from MPIE. For input admittance calculation, due to conformal current distribution on the dipole, magnetic potential DGF is zero [10].

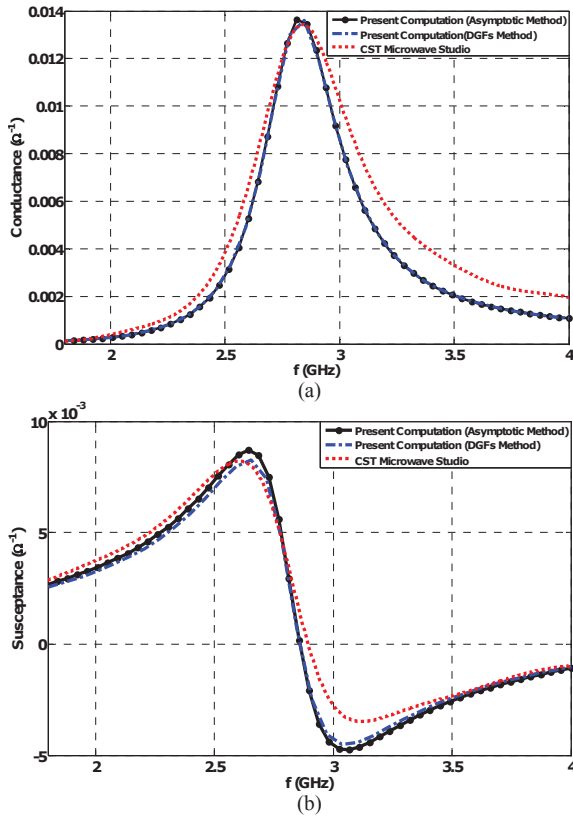


Figure 3. Input Admittance of spherical dipole antenna over PEC sphere. (a) conductance, (b) susceptance

Figure 3 illustrates the results. To show input admittance convergence speed toward spherical harmonics, the input admittance at the center frequency of the antenna versus the number of terms in spherical harmonics series is illustrated in Fig. 4. It can be noticed from this figure that the input admittance of the antenna is converged when 25 terms of spherical harmonics in the summation are used.

Electric field DGF with 30 terms of spherical harmonics is used to determine antenna radiation pattern. As it can be seen from Fig. 5, radiation pattern is directed to about 30° which is in good agreement with the results obtained from CST simulator [14]. It should be mentioned that at the shadow boundary on the conducting sphere, the incident wave excites creeping waves propagating along the sphere surface. Due to comparable dimension of the sphere to the wavelength, the creeping waves radiate surface diffracted waves [15].

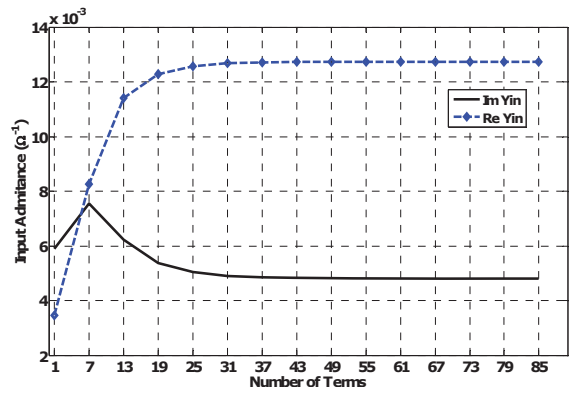


Figure 4. Convergence of Input Admittance of a spherical dipole antenna over sphere

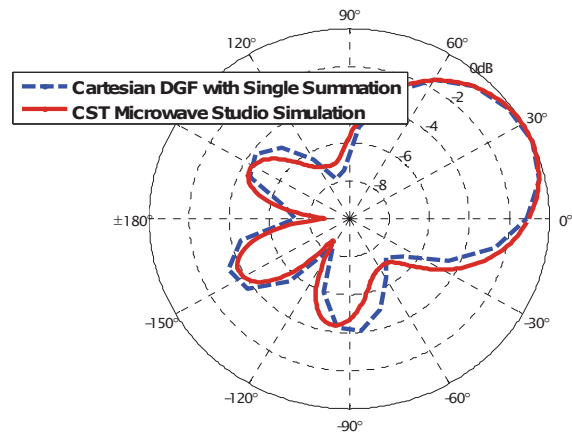


Figure 5. Radiation Pattern of a spherical dipole antenna over PEC sphere

In the following, two examples of conformal dipole antenna arrays located at the same radial distance from the coordinates center (Fig. 6) are analyzed and mutual couplings between their elements are investigated. d , defined in degrees, represents the distance between the centers of elements and can be in θ or ϕ direction.



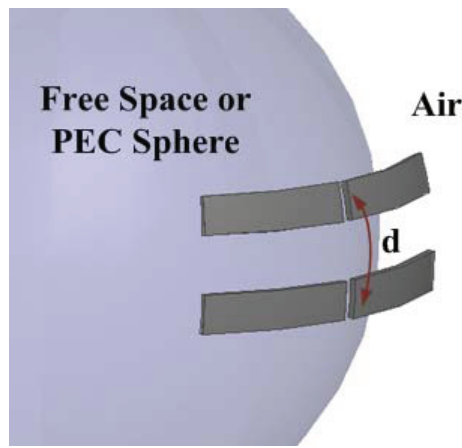


Figure 6. A pair of conformal dipole antennas

Figure 7 shows insertion loss and mutual coupling of a pair of spherical dipole antennas separated by a distance of 15° in θ direction from each other and located in free space or above a conducting sphere. Some changes in the results can be noticed from Fig.7 with and without the presence of the conducting sphere. Figure 8 demonstrates mutual coupling between two conformal dipoles over a PEC sphere for different separation distances in θ direction. Figure 9 illustrates mutual coupling between a pair of spherical dipoles located in free space or above a conducting sphere at the center frequency of the antennas versus d in ϕ direction.

Mutual couplings between the elements of an array of five conformal dipoles above a PEC sphere are also investigated. Each antenna element is separated from its adjacent element by a distance of 15° in θ direction. The lowest dipole is considered as the first element and the third dipole is located at $\theta=0^\circ$. Figure 10 shows the mutual couplings between the first three adjacent elements obtained from the presented method and Ansoft HFSS simulation.

It can be noticed that the results obtained from the presented analysis methods based on DGF or asymptotic approximation are in good agreement with the results obtained from HFSS and CST softwares. However, simulator softwares are highly dependent on meshing the structure, probe modeling and the size of radiation box. Therefore, more time and memory is required in order to obtain precise and stable results from simulator packages. The proposed methods have more calculation speed and accuracy in comparison with the mentioned softwares.

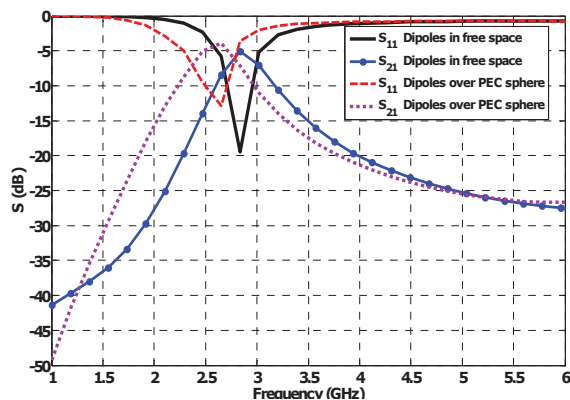


Figure 7. S-parameters of a pair of conformal dipole antennas

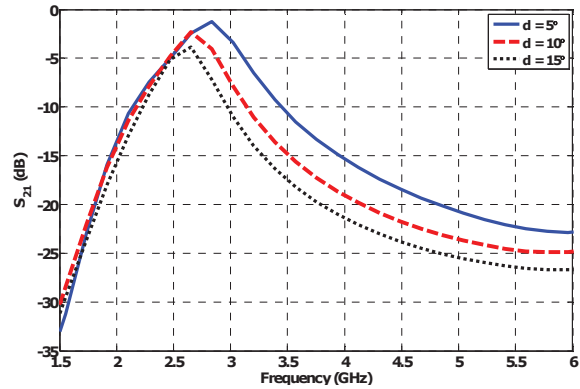


Figure 8. Mutual coupling of a pair of conformal dipole antennas over a PEC sphere

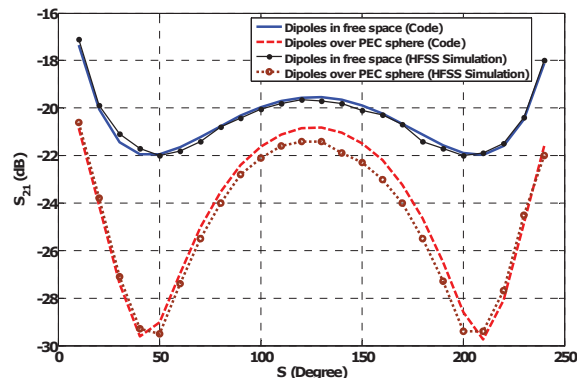


Figure 9. Mutual coupling of a pair of conformal dipole antennas versus d in ϕ direction

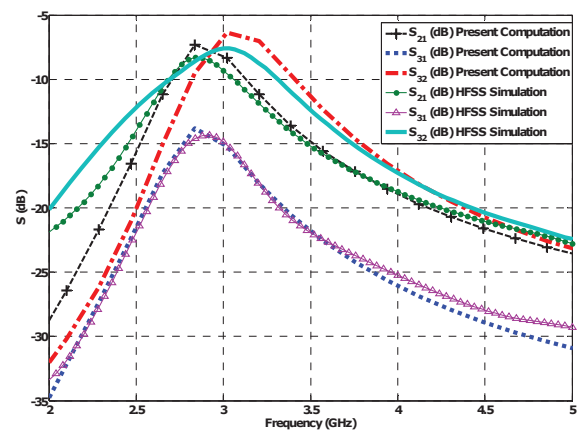


Figure 10. Mutual couplings of an array of five conformal dipole antennas over a PEC sphere

IV. CONCLUSION

In this paper, an efficient full-wave method to analyze various antennas over spherical multilayer structures has been presented. In this method, after meshing the antenna into linear triangles in Cartesian coordinates, by using MPIE formulation, the input admittance of a conformal dipole antenna over a PEC sphere has been computed. The infinite double summation has been transformed to a single summation using addition theorems for Legendre polynomials and spherical Hankel functions yielding an increase in the radiation pattern computation convergence speed. In order to determine electric field vectors at entire medium, conversion equations of a dyad from spherical to Cartesian coordinates has been



presented. To validate the proposed methods, mutual couplings between elements of a conformal dipole antenna array in free space or above a conducting sphere have been investigated. Accuracy of the proposed methods has been validated by comparing the results obtained from the presented methods with those obtained from commercial softwares.

ACKNOWLEDGEMENTS

The authors would like to thank the CyberSpace Research Institute (former Iran Telecommunication Research Center) for its financial support.

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Ahmad Hosseinbeig was born in 1983 in Tehran, Iran. He received his B.Sc. degree in electrical engineering from Shahid Bahonar University, Kerman, Iran and his M.Sc. degree in telecommunications engineering from K. N. Toosi University of Technology, Tehran, Iran in 2006 and 2008, respectively. He is presently studying

for a Ph.D. in telecommunications engineering at K. N. Toosi University of Technology, Tehran, Iran. He is currently

working on analysis and design of spherical aperture-coupled Dielectric Resonator Antennas (DRA) using dyadic Green's functions and volume surface integral equations. His main area of interest is numerical techniques in electromagnetic theory, antenna design and measurement, microwave and high frequency circuit design.



Manoochehr Kamyab received his B.Sc. and M.Sc. degree from the University of Tehran, Tehran, Iran, and his Ph.D. degree from Michigan State University, in 1982, in electrical engineering. He is a professor at the Department of Electrical Engineering, K.N. Toosi University of Technology,

Tehran, Iran. His research interests are antennas and EM modeling with dyadic Green's functions, scattering and inverse scattering problems, integrated antennas and components for broad-band wireless telecommunications, microwave and millimeter-wave circuits, antenna beam-steering techniques, millimeter and sub-millimeter-wave circuits and metamaterial applications in antennas.



Javad Soleiman Meiguni was born in 1982 in Tehran, Iran. He received his B.Sc. degree from Semnan University, Semnan, Iran and his M.Sc. degree from K. N. Toosi University of Technology, Tehran, Iran in 2005 and 2008, respectively all in electrical engineering. He has been working towards Ph.D.

degree in telecommunications at the K. N. Toosi University of Technology, Tehran, Iran. He has participated in many projects inside and outside the university. Currently, he is working on analysis and design of spherical aperture-coupled antennas using dyadic Green's functions. His research areas include antennas and EM modeling, numerical techniques in electromagnetics, antenna design and measurement, active and passive microwave and millimeter-wave circuit design.

