

# Cooperative Filter-and-Forward Beamforming in Cognitive Radio Relay Networks

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*Abstract*— In this paper, the Filter-and-Forward strategy is developed for cooperative relays in Cognitive Radio networks with underlay structure and frequency selective fading channels. A cost function is defined to minimize the required transmitting power of the relay networks and the secondary user. This is performed subject to keeping the power of noise and interferences at the primary receiver less than a predefined limit and an SINR above a given threshold. In this structure, a power control is also carried out on the secondary transmitter. Simulation results show that the proposed algorithm converges to an optimal solution based on the interior point method. The results show that compared to the Amplify-and-Forward strategy, we achieve a higher SINR threshold in the secondary receiver and need a lower transmitting power at the relays and secondary transmitter.

*Keywords*-component; cognitive radio, cooperative beamforming, Filter-and-Forward, power control, convex optimization, frequency selective channel.

## I. INTRODUCTION

Cognitive radio (CR) network is used as a key strategy to overcome the problem of radio spectrum restriction [1]. Three well-known approaches of CR are interweave, overlay, and underlay structures. Since, the underlay structure has a higher spectral efficiency [2], secondary users make use of the radio spectrum of primary users simultaneously subject to the interference leakage on them less than a specified limit [2], [3]. Thus, in this structure, the control of

interference on primary users is a challenging problem [4]. One of the main strategies for interference control is beamforming using multi-antennas in transmitters and receivers. However, due to the required power and the large size of these antennas, their use is not preferred. Instead, incorporation of relay networks has recently received increasing attention. Several signaling techniques used in relay nodes and cooperative networks are Decode and Forward (DF), Detect and Forward (DetF), and Amplify and Forward (AF) methods. On the other hand, one of the efficient

processing techniques in cooperative networks is based on beamforming [5], [6].

Cooperative beamforming in CR with the AF technique and underlay structure has recently been studied for flat fading channels. For instance, in [7], cooperative beamforming is employed to maximize the network throughput. To do so, the best relay is selected and AF beamforming is then performed by that relay. In [8], a cooperative beamforming scheme is developed in which all relay nodes are involved in beamforming and the goal is to maximize the SINR in the secondary receiver. In [9], cooperative beamforming is designed to obtain beamforming weights such that the SINR in the secondary receiver is maximized while the interference on the primary receiver is eliminated.

On the other hand, for frequency selective fading channels, the AF should be used with the OFDM which has some disadvantages like the high Peak to Average Power Ratio and carrier frequency offset [10]. To cope with such channels, the Filter-and-Forward (FF) technique has been suggested [11] in which an FIR filter is used in each relay to compensate for the channels distortion between "the transmitter and relays" and also "relays and the receiver".

In this paper, we consider a frequency selective channel and extend the FF technique for a CR network with an underlay structure. The goal is to minimize the sum of the power of relays and secondary transmitter. This strategy is interesting from the network power efficiency viewpoint. In this structure, we control the power of the secondary transmitter and perform a beamforming scheme. Also, an iterative algorithm is proposed to solve this optimization problem.

The remainder of this paper is organized as follows. In Section II, we introduce the cognitive radio network model. In Section III, we define optimization criterion and propose an algorithm for optimization. Section IV presents simulation results and finally Section V concludes the paper.

## II. COGNITIVE RADIO NETWORK MODEL

A CR network with a primary transmitter-receiver, a secondary transmitter-receiver, and  $R$  relays is shown in Fig. 1. In the 1<sup>st</sup> time slot, signals are transmitted to the relays and in the 2<sup>nd</sup> time slot relays transmit the signals to the receivers by beamforming according to a defined objective.

The received signals at the relays are defined as

$$\mathbf{r}(n) = \sum_{l=0}^{L_f-1} \sqrt{P_p} \mathbf{f}_l x^{(p)}(n-l) + \sum_{l=0}^{L'_f-1} \sqrt{P_s} \mathbf{f}'_l x^{(s)}(n-l) + \mathbf{v}(n) \quad (1)$$

where  $\mathbf{f}_l \triangleq [f_{l,1}, \dots, f_{l,R}]^T$ ,  $\mathbf{f}'_l \triangleq [f'_{l,1}, \dots, f'_{l,R}]^T$

$$\mathbf{f}_l \triangleq [f_{l,1}, \dots, f_{l,R}]^T, \mathbf{f}'_l \triangleq [f'_{l,1}, \dots, f'_{l,R}]^T$$

and  $x^{(p)}(n)$  and  $x^{(s)}(n)$  show the symbols transmitted

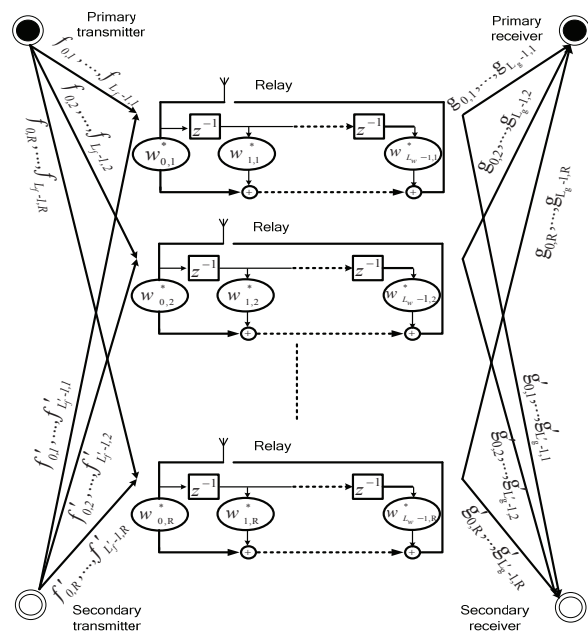


Figure 1. A cognitive radio model with FF relays.

by the primary and secondary transmitters, respectively,  $P_p$  and  $P_s$  are their respective powers, and  $\mathbf{f}_l$  and  $\mathbf{f}'_l$  are the impulse response vectors corresponding to the  $l$ -th coefficients of the channels. Also,  $f_{l,i}$ ,  $l = 0, \dots, L_f - 1$  show the channel coefficients between the primary transmitter and the  $i$ -th relay and  $f'_{l,i}$ ,  $l = 0, \dots, L'_f - 1$  denote the channel coefficients between the secondary transmitter and the  $i$ -th relay, and  $\mathbf{v}(n)$  is a zero-mean complex white Gaussian noise vector with a variance of  $\sigma_v^2$ .

By introducing the transmitted signals vectors  $\mathbf{x}^{(p)}(n)$  and  $\mathbf{x}^{(s)}(n)$ , which indicate the effect of ISI on the transmitted signals, (1) is expressed in vector form as

$$\mathbf{r}(n) = \sqrt{P_p} \mathbf{F} \mathbf{x}^{(p)}(n) + \sqrt{P_s} \mathbf{F}' \mathbf{x}^{(s)}(n) + \mathbf{v}(n) \quad (2)$$

where

$$\mathbf{F} \triangleq [\mathbf{f}_0, \dots, \mathbf{f}_{L_f-1}]_{R \times L_f}$$

$$\mathbf{F}' \triangleq [\mathbf{f}'_0, \dots, \mathbf{f}'_{L'_f-1}]_{R \times L'_f}$$

$$\mathbf{x}^{(p)}(n) \triangleq [x^{(p)}(n), \dots, x^{(p)}(n - (L_f - 1))]^T$$

$$\mathbf{x}^{(s)}(n) \triangleq [x^{(s)}(n), \dots, x^{(s)}(n - (L'_f - 1))]^T.$$

Accordingly, the transmitted signals vector of relays is given by

$$\mathbf{y}(n) = \sum_{l=0}^{L_f-1} \mathbf{W}_l^H (\sqrt{P_p} \mathbf{F} \mathbf{x}^{(p)}(n-l) + \sqrt{P_s} \mathbf{F}' \mathbf{x}^{(s)}(n-l) + \mathbf{v}(n-l)) \quad (3)$$

where  $\mathbf{W}_l$  is a diagonal matrix containing beamforming coefficients corresponding to the  $l$ -th coefficients of FIR filters. By including the effect of FIR filters on  $\mathbf{x}^{(p)}(n)$  and  $\mathbf{x}^{(s)}(n)$ , (3) is expressed as



$$\mathbf{y}(n) = \sum_{l=0}^{L_g-1} \mathbf{W}_l^H (\sqrt{P_p} \mathbf{F}_l \tilde{\mathbf{x}}^{(p)}(n) + \sqrt{P_s} \mathbf{F}'_l \tilde{\mathbf{x}}^{(s)}(n) + \mathbf{v}(n-l)) \quad (4)$$

where

$$\begin{aligned} \tilde{\mathbf{x}}^{(p)}(n) &\triangleq [x^{(p)}(n), \dots, x^{(p)}(n - (L_f + L_w - 2))]^T \\ \tilde{\mathbf{x}}^{(s)}(n) &\triangleq [x^{(s)}(n), \dots, x^{(s)}(n - (L'_f + L_w - 2))]^T \\ \mathbf{F}_l &\triangleq [\underbrace{\mathbf{0}_{R \times 1}, \dots, \mathbf{0}_{R \times 1}}_{R \times l}, \underbrace{\mathbf{F}_l}_{R \times L_f}, \underbrace{\mathbf{0}_{R \times 1}, \dots, \mathbf{0}_{R \times 1}}_{R \times L_w - l - 1}] \\ \mathbf{F}'_l &\triangleq [\underbrace{\mathbf{0}_{R \times 1}, \dots, \mathbf{0}_{R \times 1}}_{R \times l}, \underbrace{\mathbf{F}'_l}_{R \times L'_f}, \underbrace{\mathbf{0}_{R \times 1}, \dots, \mathbf{0}_{R \times 1}}_{R \times L_w - l - 1}] \end{aligned}$$

in which  $\mathbf{0}_{R \times 1}$  is a  $R \times 1$  zero matrix. To simplify computations, the following definitions are used.

$$\begin{aligned} \mathbf{\Xi} &\triangleq [\mathbf{F}_0^T, \dots, \mathbf{F}_{L_g-1}^T]^T \\ \mathbf{\Xi}' &\triangleq [\mathbf{F}'_0^T, \dots, \mathbf{F}'_{L_g-1}^T]^T \\ \mathbf{W} &\triangleq [\underbrace{\mathbf{W}_0}_{R \times R}, \dots, \underbrace{\mathbf{W}_{L_g-1}}_{R \times R}]^T \\ \tilde{\mathbf{v}}(n) &\triangleq [\mathbf{v}^T(n), \dots, \mathbf{v}^T(n - (L_w - 1))]^T_{RL_w \times 1} \end{aligned}$$

$$\mathbf{y}(n) = \sqrt{P_p} \mathbf{W}^H \mathbf{\Xi} \tilde{\mathbf{x}}^{(p)}(n) + \sqrt{P_s} \mathbf{W}^H \mathbf{\Xi}' \tilde{\mathbf{x}}^{(s)}(n) + \mathbf{W}^H \tilde{\mathbf{v}}(n). \quad (5)$$

The received signals by the primary and secondary receivers are expressed respectively as

$$\begin{aligned} z^{(p)}(n) &= \sum_{l=0}^{L_g-1} \mathbf{g}_l^T (\sqrt{P_p} \mathbf{W}^H \mathbf{\Xi} \tilde{\mathbf{x}}^{(p)}(n-l) \\ &+ \sqrt{P_s} \mathbf{W}^H \mathbf{\Xi}' \tilde{\mathbf{x}}^{(s)}(n-l) + \mathbf{W}^H \tilde{\mathbf{v}}(n-l)) + v^{(p)}(n) \end{aligned} \quad (6)$$

$$\begin{aligned} z^{(s)}(n) &= \sum_{l=0}^{L'_g-1} \mathbf{g}'_l{}^T (\sqrt{P_p} \mathbf{W}^H \mathbf{\Xi} \tilde{\mathbf{x}}^{(p)}(n-l) \\ &+ \sqrt{P_s} \mathbf{W}^H \mathbf{\Xi}' \tilde{\mathbf{x}}^{(s)}(n-l) + \mathbf{W}^H \tilde{\mathbf{v}}(n-l)) + v^{(s)}(n) \end{aligned} \quad (7)$$

where  $\mathbf{g}_l \triangleq [g_{l,1}, \dots, g_{l,R}]^T$  and  $\mathbf{g}'_l \triangleq [g'_{l,1}, \dots, g'_{l,R}]^T$  are the impulse response vectors corresponding to the  $l$ -th coefficients of the channels between the relays and the receivers and  $v^{(p)}(n)$  and  $v^{(s)}(n)$  show the noise of the primary and secondary receivers with variances  $\sigma_{v^{(s)}}^2$  and  $\sigma_{v^{(p)}}^2$ , respectively. Using the Kronecker multiplication properties and  $\mathbf{a}^T \text{diag}(\mathbf{b}) = \mathbf{b}^T \text{diag}(\mathbf{a})$ , we can write

$$\begin{aligned} \mathbf{g}_l^T \mathbf{W}^H &= [\mathbf{g}_l^T \mathbf{W}_0^H, \dots, \mathbf{g}_l^T \mathbf{W}_{L_g-1}^H] \\ &= [\boldsymbol{\omega}_0^H \mathbf{G}_l, \dots, \boldsymbol{\omega}_{L_g-1}^H \mathbf{G}_l] = \boldsymbol{\omega}^H (\mathbf{I}_{L_g} \otimes \mathbf{G}_l) \\ \mathbf{g}'_l{}^T \mathbf{W}^H &= \boldsymbol{\omega}'^H (\mathbf{I}_{L'_g} \otimes \mathbf{G}'_l), \\ \mathbf{G}_l &\triangleq \text{diag} \{ \mathbf{g}_l \}, \mathbf{G}'_l \triangleq \text{diag} \{ \mathbf{g}'_l \}, \boldsymbol{\omega}_l \triangleq \text{diag} \{ \mathbf{W}_l \} \end{aligned}$$

where  $\mathbf{I}_{L_g}$  is an  $L_g \times L_g$  identity matrix,  $\otimes$  represents the Kronecker product, and  $\text{diag}(\mathbf{A})$  shows a vector

of the diagonal elements matrix  $\mathbf{A}$ . Then, (6) and (7) are respectively presented as

$$\begin{aligned} z^{(p)}(n) &= \sum_{l=0}^{L_g-1} (\sqrt{P_p} \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \mathbf{\Xi} \tilde{\mathbf{x}}^{(p)}(n-l) \\ &+ \sqrt{P_s} \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \mathbf{\Xi}' \tilde{\mathbf{x}}^{(s)}(n-l) \\ &+ \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \tilde{\mathbf{v}}(n-l)) + v^{(p)}(n) \end{aligned} \quad (8)$$

$$\begin{aligned} z^{(s)}(n) &= \sum_{l=0}^{L'_g-1} (\sqrt{P_p} \boldsymbol{\omega}'^H (\mathbf{I}_{L_w} \otimes \mathbf{G}'_l) \mathbf{\Xi} \tilde{\mathbf{x}}^{(p)}(n-l) \\ &+ \sqrt{P_s} \boldsymbol{\omega}'^H (\mathbf{I}_{L_w} \otimes \mathbf{G}'_l) \mathbf{\Xi}' \tilde{\mathbf{x}}^{(s)}(n-l) \\ &+ \boldsymbol{\omega}'^H (\mathbf{I}_{L_w} \otimes \mathbf{G}'_l) \tilde{\mathbf{v}}(n-l)) + v^{(s)}(n). \end{aligned} \quad (9)$$

To simplify (8) and (9), the effect of the frequency selective channels on the relay transmitted signals and noise are defined as

$$\begin{aligned} \tilde{\tilde{\mathbf{x}}}^{(p)}(n) &\triangleq [x^{(p)}(n), \dots, x^{(p)}(n - (L_f + L_w + L_g - 3))]^T \\ \tilde{\tilde{\mathbf{x}}}^{(s)}(n) &\triangleq [x^{(s)}(n), \dots, x^{(s)}(n - (L'_f + L_w + L_g - 3))]^T \\ \tilde{\tilde{\mathbf{x}}}^{(p)}(n) &\triangleq [x^{(p)}(n), \dots, x^{(p)}(n - (L_f + L_w + L'_g - 3))]^T \\ \tilde{\tilde{\mathbf{x}}}^{(s)}(n) &\triangleq [x^{(s)}(n), \dots, x^{(s)}(n - (L'_f + L_w + L'_g - 3))]^T \\ \tilde{\tilde{\mathbf{v}}}(n) &\triangleq [\mathbf{v}^T(n), \dots, \mathbf{v}^T(n - (L_w + L_g - 2))]^T \\ \tilde{\tilde{\tilde{\mathbf{v}}}}(n) &\triangleq [\mathbf{v}^T(n), \dots, \mathbf{v}^T(n - (L_w + L'_g - 2))]^T \\ \tilde{\mathbf{I}}_l &\triangleq [\underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times l}, \underbrace{\mathbf{I}_{RL_w}}_{RL_w \times RL_w}, \underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times (L_g - l - 1)}] \\ \tilde{\mathbf{\Xi}}_l &\triangleq [\underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times l}, \underbrace{\mathbf{\Xi}}_{RL_w \times (L_f + L_w - 1)}, \underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times (L_g - l - 1)}] \\ \tilde{\mathbf{\Xi}}'_l &\triangleq [\underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times l}, \underbrace{\mathbf{\Xi}'}_{RL_w \times (L'_f + L_w - 1)}, \underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times (L_g - l - 1)}] \\ \tilde{\mathbf{\Xi}}''_l &\triangleq [\underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times l}, \underbrace{\mathbf{\Xi}''}_{RL_w \times (L_f + L_w - 1)}, \underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times (L_g - l - 1)}] \\ \tilde{\mathbf{\Xi}}'''_l &\triangleq [\underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times l}, \underbrace{\mathbf{\Xi}'''}_{RL_w \times (L'_f + L_w - 1)}, \underbrace{\mathbf{0}_{RL_w \times 1}, \dots, \mathbf{0}_{RL_w \times 1}}_{RL_w \times (L'_g - l - 1)}] \end{aligned}$$

where  $\mathbf{I}_{RL_w}$  is an  $RL_w \times RL_w$  identity matrix. Then, (8) and (9) are expressed as

$$\begin{aligned} z^{(p)}(n) &= \sum_{l=0}^{L_g-1} (\sqrt{P_p} \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \tilde{\mathbf{\Xi}}_l \tilde{\tilde{\mathbf{x}}}^{(p)}(n) \\ &+ \sqrt{P_s} \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \tilde{\mathbf{\Xi}}'_l \tilde{\tilde{\mathbf{x}}}^{(s)}(n) \\ &+ \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \tilde{\mathbf{I}}_l \tilde{\tilde{\mathbf{v}}}(n)) + v^{(p)}(n) \end{aligned} \quad (10)$$

$$\begin{aligned} z^{(s)}(n) &= \sum_{l=0}^{L'_g-1} (\sqrt{P_p} \boldsymbol{\omega}'^H (\mathbf{I}_{L_w} \otimes \mathbf{G}'_l) \tilde{\mathbf{\Xi}}''_l \tilde{\tilde{\mathbf{x}}}^{(p)}(n) \\ &+ \sqrt{P_s} \boldsymbol{\omega}'^H (\mathbf{I}_{L_w} \otimes \mathbf{G}'_l) \tilde{\mathbf{\Xi}}'''_l \tilde{\tilde{\mathbf{x}}}^{(s)}(n) \\ &+ \boldsymbol{\omega}'^H (\mathbf{I}_{L_w} \otimes \mathbf{G}'_l) \tilde{\mathbf{I}}'_l \tilde{\tilde{\tilde{\mathbf{v}}}}(n)) + v^{(s)}(n) \end{aligned} \quad (11)$$



To remove out the summation operator from (10) and (11) for more simplicity, we substitute the following term

$$\sum_{l=0}^{L_g-1} (\mathbf{I}_{L_w} \otimes \mathbf{G}_l) \boldsymbol{\Xi}_l = \mathbf{G}\tilde{\mathbf{F}}$$

which

$$\begin{aligned} \mathbf{G} &\triangleq [\mathbf{I}_{L_w} \otimes \mathbf{G}_0, \dots, \mathbf{I}_{L_w} \otimes \mathbf{G}_{L_g-1}]_{RL_w \times RL_w L_g} \\ \tilde{\mathbf{F}} &\triangleq [\boldsymbol{\Xi}_0^T, \dots, \boldsymbol{\Xi}_{L_g-1}^T]^T \\ \bar{\mathbf{F}} &\triangleq [\bar{\mathbf{f}}, \bar{\mathbf{F}}] \\ \tilde{\mathbf{I}} &\triangleq [\tilde{\mathbf{I}}_0^T, \dots, \tilde{\mathbf{I}}_{L_g-1}^T]^T_{RL_w L_g \times R(L_g + L_w - 1)} \end{aligned}$$

Then, by separating the desired signals of the primary and secondary users from the ISI and noise components, we get

$$\begin{aligned} z^{(p)}(n) &= \sqrt{P_p} \boldsymbol{\omega}^H \mathbf{G} [\bar{\mathbf{f}}, \bar{\mathbf{F}}] \begin{bmatrix} x^{(p)}(n) \\ \bar{\mathbf{x}}^{(p)}(n) \end{bmatrix} \\ &+ \sqrt{P_s} \boldsymbol{\omega}^H \mathbf{G}\tilde{\mathbf{F}}' \tilde{\mathbf{x}}^{(s)}(n) + \boldsymbol{\omega}^H \mathbf{G}\tilde{\mathbf{I}} \tilde{\mathbf{v}}(n) + v^{(p)}(n) \\ &= \underbrace{\sqrt{P_p} \boldsymbol{\omega}^H \mathbf{G}\bar{\mathbf{f}} x^{(p)}(n)}_{\text{Desired signal component}} + \underbrace{\sqrt{P_p} \boldsymbol{\omega}^H \mathbf{G}\bar{\mathbf{F}} \bar{\mathbf{x}}^{(p)}(n)}_{\text{ISI component}} \\ &+ \underbrace{\sqrt{P_s} \boldsymbol{\omega}^H \mathbf{G}\tilde{\mathbf{F}}' \tilde{\mathbf{x}}^{(s)}(n)}_{\text{Interference from secondary transmitter component}} + \underbrace{\boldsymbol{\omega}^H \mathbf{G}\tilde{\mathbf{I}} \tilde{\mathbf{v}}(n) + v^{(p)}(n)}_{\text{Noise component}} \end{aligned} \quad (12)$$

$$\begin{aligned} z^{(s)}(n) &= \underbrace{\sqrt{P_s} \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{f}}'' x^{(s)}(n)}_{\text{Desired signal component}} + \underbrace{\sqrt{P_s} \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{F}}'' \tilde{\mathbf{x}}^{(s)}(n)}_{\text{ISI component}} \\ &+ \underbrace{\sqrt{P_p} \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{F}}'' \tilde{\mathbf{x}}^{(p)}(n)}_{\text{Interference from primary transmitter component}} + \underbrace{\boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{I}}'' \tilde{\mathbf{v}}(n) + v^{(s)}(n)}_{\text{Noise component}} \end{aligned} \quad (13)$$

in which the structures of  $\mathbf{G}\tilde{\mathbf{F}}'$ ,  $\mathbf{G}'\tilde{\mathbf{F}}''$ ,  $\mathbf{G}'\tilde{\mathbf{F}}'''$ ,  $\mathbf{G}'$ ,  $\tilde{\mathbf{F}}'$ ,  $\tilde{\mathbf{F}}''$ ,  $\tilde{\mathbf{F}}'''$ , and  $\tilde{\mathbf{I}}$  are defined similar to those of  $\mathbf{G}\tilde{\mathbf{F}}$ ,  $\mathbf{G}$ ,  $\tilde{\mathbf{F}}$ , and  $\tilde{\mathbf{I}}$ .

### III. POWER MINIMIZATION

To compute the beamforming vector, containing the FIR filters coefficients and the secondary transmitted power, an objective function is defined as

$$\begin{aligned} \min_{P_s, \boldsymbol{\omega}} \quad & P_T + P_s \\ \text{s.t.} \quad & \text{SINR}^{(s)} \geq \gamma \\ & \eta \leq \xi \end{aligned} \quad (14)$$

where  $P_T$  is the total relays transmitted powers,  $P_s$  is the secondary transmitted power,  $\text{SINR}^{(s)}$  is the SINR in the secondary receiver,  $\gamma$  is the SINR threshold,  $\eta$  is the total interference and noise power on the primary receiver, and  $\xi$  is the interference limit.

The individual transmit power of the  $i$ -th relay is obtained using (4) and incorporation of (14) as

$$\begin{aligned} P_i &= E(|y_i|^2) = P_p \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{E}_i) \boldsymbol{\Xi} \boldsymbol{\Xi}^H (\mathbf{I}_{L_w} \otimes \mathbf{E}_i)^H \boldsymbol{\omega} \\ &+ P_s \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{E}_i) \boldsymbol{\Xi}' \boldsymbol{\Xi}'^H (\mathbf{I}_{L_w} \otimes \mathbf{E}_i)^H \boldsymbol{\omega} \\ &+ \sigma_v^2 \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{E}_i) (\mathbf{I}_{L_w} \otimes \mathbf{E}_i)^H \boldsymbol{\omega} \end{aligned} \quad (15)$$

where

$$\begin{aligned} \mathbf{e}_i^T \mathbf{W}^H &= [\mathbf{e}_i^T \mathbf{W}_0^H, \dots, \mathbf{e}_i^T \mathbf{W}_{L_w-1}^H] \\ &= [\boldsymbol{\omega}_0^H \mathbf{E}_i, \dots, \boldsymbol{\omega}_{L_w-1}^H \mathbf{E}_i]^T = \boldsymbol{\omega}^H (\mathbf{I}_{L_w} \otimes \mathbf{E}_i) \end{aligned}$$

and  $\mathbf{e}_i$  is the  $i$ -th column of the identity matrix and  $\mathbf{E}_i = \text{diag}(\mathbf{e}_i)$ .

Then, the total relays transmit power is given by

$$P_T = \sum_{i=1}^R P_i = \boldsymbol{\omega}^H \left( \sum_{i=1}^R \mathbf{D}_i + P_s \mathbf{D}' \right) \boldsymbol{\omega} = \boldsymbol{\omega}^H (\mathbf{D} + P_s \mathbf{D}') \boldsymbol{\omega} \quad (16)$$

The power of the desired signal of the secondary receiver is obtained from (13) as

$$\begin{aligned} P_s^{(s)} &= E \left( \left| \sqrt{P_s} \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{f}}'' x^{(s)}(n) \right|^2 \right) \\ &= P_s \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{f}}'' \tilde{\mathbf{f}}''^H \mathbf{G}'^H \boldsymbol{\omega} = P_s \boldsymbol{\omega}^H \mathbf{Q}_s^{(s)} \boldsymbol{\omega} \end{aligned} \quad (17)$$

The interference power at the secondary receiver, which is the sum of the ISI and the interference caused by the primary transmitter, is obtained from (13) as

$$\begin{aligned} P_i^{(s)} &= E \left( \left| \sqrt{P_s} \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{F}}'' \tilde{\mathbf{x}}^{(s)}(n) + \sqrt{P_p} \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{F}}'' \tilde{\mathbf{x}}^{(p)}(n) \right|^2 \right) \\ &= P_s \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{F}}'' \tilde{\mathbf{F}}''^H \mathbf{G}'^H \boldsymbol{\omega} + P_p \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{F}}'' \tilde{\mathbf{F}}''^H \mathbf{G}'^H \boldsymbol{\omega} \\ &= P_s \boldsymbol{\omega}^H \mathbf{Q}_{I(s)}^{(s)} \boldsymbol{\omega} + \boldsymbol{\omega}^H \mathbf{Q}_{I(p)}^{(s)} \boldsymbol{\omega} \end{aligned} \quad (18)$$

where  $\mathbf{Q}_{I(p)}^{(s)}$  and  $\mathbf{Q}_{I(s)}^{(s)}$  indicate the interference from the primary transmitter and the ISI, respectively. Also, from (13), the noise power in the secondary receiver is given by

$$\begin{aligned} P_n^{(s)} &= E \left( \left| \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{I}}'' \tilde{\mathbf{v}}(n) + v^{(s)}(n) \right|^2 \right) \\ &= \sigma_v^2 \boldsymbol{\omega}^H \mathbf{G}' \tilde{\mathbf{I}}'' \tilde{\mathbf{I}}''^H \mathbf{G}'^H \boldsymbol{\omega} + \sigma_v^2 = \boldsymbol{\omega}^H \mathbf{Q}_n^{(s)} \boldsymbol{\omega} + \sigma_v^2 \end{aligned} \quad (19)$$

In a similar manner, the interference power in the primary receiver is defined as

$$\begin{aligned} P_i^{(p)} &= E \left( \left| \sqrt{P_p} \boldsymbol{\omega}^H \mathbf{G}\bar{\mathbf{F}} \bar{\mathbf{x}}^{(p)}(n) + \sqrt{P_s} \boldsymbol{\omega}^H \mathbf{G}\tilde{\mathbf{F}}' \tilde{\mathbf{x}}^{(s)}(n) \right|^2 \right) \\ &= P_p \boldsymbol{\omega}^H \mathbf{G}\bar{\mathbf{F}} \bar{\mathbf{F}}^H \mathbf{G}^H \boldsymbol{\omega} + P_s \boldsymbol{\omega}^H \mathbf{G}\tilde{\mathbf{F}}' \tilde{\mathbf{F}}'^H \mathbf{G}^H \boldsymbol{\omega} \\ &= \boldsymbol{\omega}^H \mathbf{Q}_{I(p)}^{(p)} \boldsymbol{\omega} + P_s \boldsymbol{\omega}^H \mathbf{Q}_{I(s)}^{(p)} \boldsymbol{\omega} \end{aligned} \quad (20)$$



while  $\mathbf{Q}_{I(p)}^{(p)}$  and  $\mathbf{Q}_{I(s)}^{(p)}$  are the interferences caused by the ISI and the secondary transmitter, respectively. The noise power in the primary receiver is given by

$$P_{n}^{(p)} = E\left(\left|\omega^H \mathbf{G} \tilde{\mathbf{v}}(n) + v^{(p)}(n)\right|^2\right) = \sigma_v^2 \omega^H \mathbf{G} \tilde{\mathbf{H}}^H \mathbf{G}^H \omega + \sigma_{v^{(p)}}^2 = \omega^H \mathbf{Q}_v^{(p)} \omega + \sigma_{v^{(p)}}^2 \quad (21)$$

where  $\mathbf{Q}_v^{(p)}$  is related to the additive noise at relays.

Using (19)-(21), (14) is reformulated as

$$\begin{aligned} \min_{\omega, P_s} \quad & P_s + \omega^H \mathbf{D} \omega + P_s \omega^H \mathbf{D}' \omega \\ s.t. \quad & \frac{P_s \omega^H \mathbf{Q}_s^{(s)} \omega}{P_s \omega^H \mathbf{Q}_{I(s)}^{(s)} \omega + \omega^H \mathbf{Q}_{I(p)}^{(s)} \omega + \omega^H \mathbf{Q}_v^{(s)} \omega + \sigma_{v^{(s)}}^2} \geq \gamma \\ & \omega^H \mathbf{Q}_{I(p)}^{(p)} \omega + P_s \omega^H \mathbf{Q}_{I(s)}^{(p)} \omega + \omega^H \mathbf{Q}_v^{(p)} \omega + \sigma_{v^{(p)}}^2 \leq \xi \end{aligned} \quad (22)$$

Next, to solve (22), we intend to change the non-convex model of (22) to a convex form. In so doing, an iterative algorithm is developed by considering a fixed secondary transmitted power  $P_s$ , and optimizing the beamformer coefficients vector. In this way, by using an auxiliary variable defined as  $\mathbf{\Gamma} = \omega \omega^H$ , (22) is stated as

$$\begin{aligned} \min_{\mathbf{\Gamma}} \quad & P_s + \text{tr}(\mathbf{\Gamma} \mathbf{D}) + P_s \text{tr}(\mathbf{\Gamma} \mathbf{D}') \\ s.t. \quad & \text{tr}(\mathbf{\Gamma} (P_s \mathbf{Q}_s^{(s)} - \gamma (P_s \mathbf{Q}_{I(s)}^{(s)} + \mathbf{Q}_{I(p)}^{(s)} + \mathbf{Q}_v^{(s)}))) \geq \gamma \sigma_{v^{(s)}}^2 \\ & \text{tr}(\mathbf{\Gamma} (\mathbf{Q}_{I(p)}^{(p)} + P_s \mathbf{Q}_{I(s)}^{(p)} + \mathbf{Q}_v^{(p)})) \leq \xi - \sigma_{v^{(p)}}^2 \\ & \text{rank}(\mathbf{\Gamma}) = 1, \mathbf{\Gamma} \geq 0 \end{aligned} \quad (23)$$

where  $\text{tr}(\cdot)$  shows the trace operator. In the preceding relationship, due to the symmetry and Positive Semi-Definite property of  $\mathbf{\Gamma}$ , the cost function and all of its constraints except the constraint  $\text{rank}(\mathbf{\Gamma}) = 1$  are convex. To have a complete convex for (23), we relax the constraint  $\text{rank}(\mathbf{\Gamma}) = 1$  to obtain a convex form [12]. The latter problem is then solved by the interior point method which is an efficient and reliable solution. A well-used MATLAB file for numerical solution of this problem is CVX [13]. After computing  $\mathbf{\Gamma}(i)$  ( $\mathbf{\Gamma}$  at the  $i$ -th iteration) for a fixed  $P_s(i-1)$  ( $P_s$  at the  $(i-1)$ -th iteration), if the  $\text{rank}(\mathbf{\Gamma}(i)) = 1$ , the beamforming coefficients vector  $\omega(i)$  ( $\omega$  at the  $i$ -th iteration) is equal to the normalized eigenvector corresponding to nonzero eigenvalue of  $\mathbf{\Gamma}(i)$ . Otherwise, one of the randomization methods in [14] must be used. Using the Lagrange and dual Lagrange method, we can prove that the relaxation problem and original problem have the same responses. This comes from the fact that the duality gap between the relaxed and original problems is zero and our problem converges to an optimal value [15].

Then, optimization should be performed with respect to  $P_s$  for the estimated  $\omega(i)$  as

$$\begin{aligned} \min_{P_s} \quad & P_s + \omega^H(i) \mathbf{D} \omega(i) + P_s \omega^H(i) \mathbf{D}' \omega(i) \\ s.t. \quad & P_s \geq \frac{\gamma \sigma_{v^{(s)}}^2 + \gamma \omega^H(i) \mathbf{Q}_{I(p)}^{(s)} \omega(i) + \gamma \omega^H(i) \mathbf{Q}_v^{(s)} \omega(i)}{\omega^H(i) \mathbf{Q}_s^{(s)} \omega(i) - \gamma \omega^H(i) \mathbf{Q}_{I(s)}^{(s)} \omega(i)} \\ & P_s \leq \frac{\xi - \sigma_{v^{(p)}}^2 - \omega^H(i) \mathbf{Q}_{I(p)}^{(p)} \omega(i) - \omega^H(i) \mathbf{Q}_v^{(p)} \omega(i)}{\omega^H(i) \mathbf{Q}_{I(s)}^{(p)} \omega(i)} \end{aligned} \quad (24)$$

In this way, minimization with respect to  $P_s$  has a linear form and its solution for a feasible case is given by an iterative relationship as

$$P_s(i) = \frac{\gamma \sigma_{v^{(s)}}^2 + \gamma \omega^H(i) \mathbf{Q}_{I(p)}^{(s)} \omega(i) + \gamma \omega^H(i) \mathbf{Q}_v^{(s)} \omega(i)}{\omega^H(i) \mathbf{Q}_s^{(s)} \omega(i) - \gamma \omega^H(i) \mathbf{Q}_{I(s)}^{(s)} \omega(i)} \quad (25)$$

As a convergence criterion, (25) stops when  $P_s(i-1) - P_s(i) \leq \varepsilon$ .

In the proposed algorithm, the computational complexity is  $O((RL_w)^4(RL_w + 2)^{2.5})$  and  $8(RL_w)^2 + 4(RL_w) + 3$  for the beamforming optimization and power allocation problems, respectively. Therefore, each step of the algorithm requires the number of  $O((RL_w)^4(RL_w + 2)^{2.5} + 8(RL_w)^2 + 4RL_w)$  operations.

#### IV. SIMULATION RESULTS

We define the channels impulse response coefficients between transmitters-relays-receivers as complex Gaussian random variables with zero mean and exponential power delay profile as [11]

$$p(t) = \frac{1}{\sigma_t} \sum_{l=0}^{L_x-1} e^{-t/\sigma_t} \delta(t - lT_s) \quad (26)$$

where  $L_x \in \{L_f, L_g, L'_f, L'_g\}$  and  $L_f = L_g = L'_f = L'_g = 5$ ,  $T_s$  is the symbol length, and  $\sigma_t = 2T_s$  denotes the delay spread. The noise variances is 0.1, the number of relays is 10, and  $\varepsilon = 0.01$ .

In the first experiment, the convergence behavior of the proposed algorithm is presented for different filters lengths. The SINR threshold is  $\gamma = 5dB$  and the interference limit is  $\xi = 0dBw$ . Note that for the filter length  $L_w = 1$ , the FF is equivalent to the AF. As seen in Fig. 2, the curves converge to their optimal values after some iterations. Meanwhile, by increasing the FIR filter length, the minimum sum of the total relays transmitted power and the secondary transmitter power decreases. Moreover, the AF ( $L_w = 1$ ) total power is much higher than that of the FF structure. Note that in our simulations, we have never dealt with the case that the rank of  $\mathbf{\Gamma}$  is higher than one. Hence, we use the normalized eigenvector for  $\omega$ .





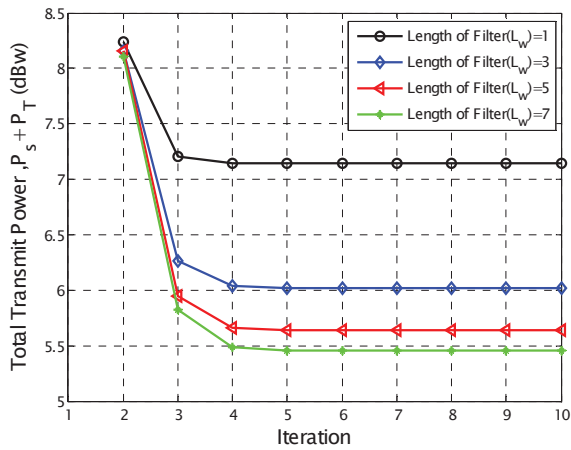


Figure 2. Convergence behaviour of the proposed algorithm for different filters lengths.

In Fig. 3, the minimum total power ( $P_s + P_T$ ) versus the SINR threshold is shown. As seen, the transmitted power is decreased for the FF case compared to the AF. Also, by increasing the SINR threshold in the secondary receiver, the total transmitted power is increased. In addition, in Fig. 4, we see that by increasing the filter length, the probability of feasibility is increased and a higher SINR threshold is achieved. A problem is called feasible, if it is solvable for more than a half of simulation runs. Otherwise, it is infeasible and the corresponding points are discarded [11].

In Figs .5 and 6, we inspect the effect of interference limit on the total transmit power. We consider the filter lengths of 1 and 5. As observed, the FF needs a lower power compared to the AF ( $L_w = 1$ ) and also achieves a higher SINR threshold in the secondary receiver. Furthermore, by increasing the interference limit, a higher SINR threshold is achieved.

V. CONCLUSION

A Filter-and-Forward strategy was applied to cooperative CR networks with an underlay structure. A cost function was defined subject to keeping the power of noise and interferences on the primary receiver less than a predefined limit, and the SINR above a given threshold. An iterative algorithm based on the interior point method was developed. Using simulation results, it was shown that the proposed algorithm compared to the AF strategy achieves a higher SINR threshold in the secondary receiver and needs a lower transmitted power in relays and the secondary transmitter.

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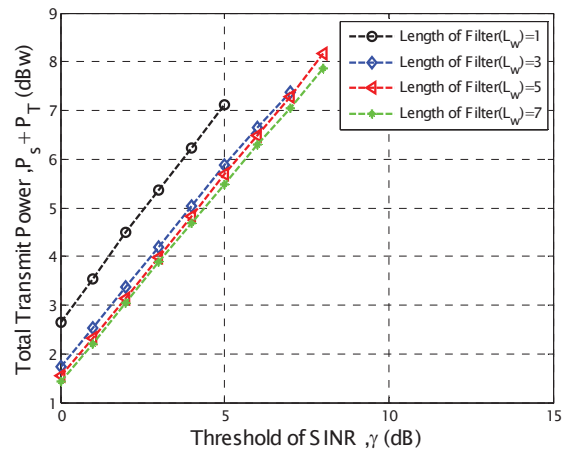


Figure 3. Minimum sum of total relays transmitted power and secondary transmitter power versus the SINR threshold for various filters lengths.

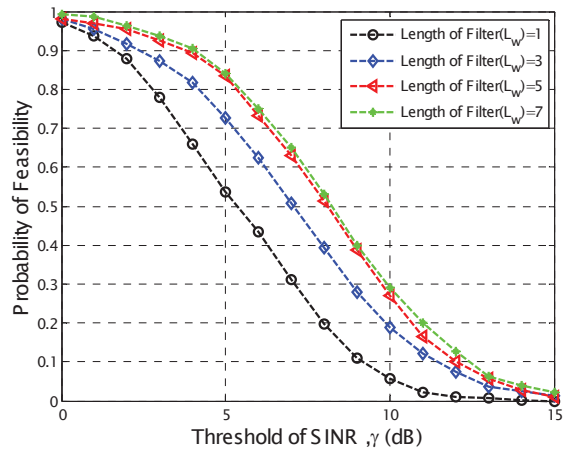


Figure 4. Feasibility probability versus SINR threshold in the secondary receiver for various filters lengths.

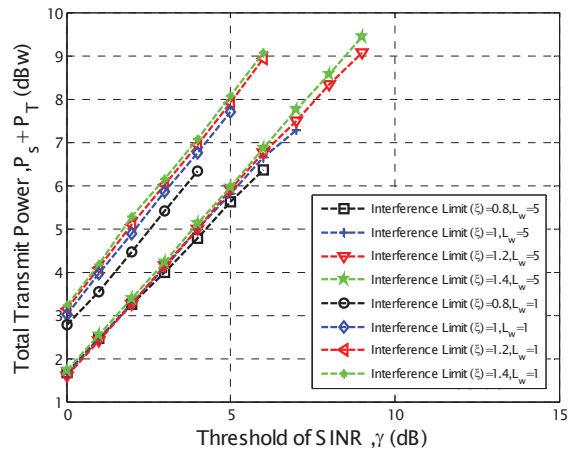


Figure 5. Minimum sum of total relays transmitted power and secondary transmitter power versus SINR threshold for various values of the interference limit.



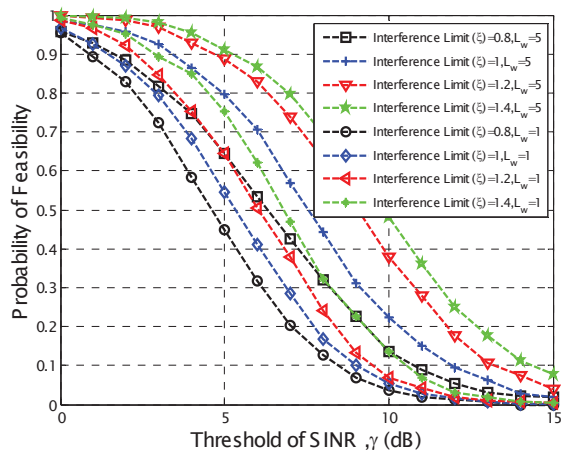


Figure 6. Feasibility probability versus SINR threshold in secondary receiver for various values interference limit.

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