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Dimensioning of 5G Networks by Using Stochastic Geometry

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Abstract—**In this paper, we propose an analytical model for dimensioning of Orthogonal Frequency Division Multiple Access (OFDMA) systems in 5G networks by considering Internet of Thing (IoT) application using stochastic geometry. In these systems, some communication is lost when the number of required subcarriers is greater than the number of the available subcarriers. We compute the upper bound of the lost communication probability for downlink. In such a system, the position of the receiving users is modeled by the Poisson point process (PPP). The number of subcarriers dedicated to each user depends on its Signal to Noise Ratio (SNR), position and the shadowing, hence for calculating the number of subcarriers, it is needed to use stochastic geometry. Since the focus of our work is on IoT application in 5G networks, a multi-group user system with each group of users having its own application and throughput requirement is considered. For having dimensioning in terms of subcarriers, we present concentration inequality for functions defined on PPP to calculate the upper bound of loss probability. The performance of the upper bound in different range of user intensity is investigated.**

Keywords—**OFDMA; dimensioning; stochastic geometry**

I. Introduction

Future fifth generation (5G) cellular standards cover new emerging technologies such as cognitive radio, Device to Device (D2D), energy harvesting, massive Multiple-Input Multiple-Output (mMIMO), and millimeter wave networks and each technology has its own specific demand for data traffic, service latency, and control overhead. In general, we consider the 5G network as a multi-class system in which each class of technology has its own traffic pattern and throughput requirement. One of the classes is IoT networks. IoT is an emerging technology which revolutionize the communication network by connecting the smart devices to the internet. In IoT application, several groups of devices are considered which each group of them requires its own bandwidth and power. The recent standards for the cellular networks like Long Term Evolution (LTE) does not work well for low data rate and low power devices in the IoT networks. One of the purposes of the 5G is to address the existing limits of the previous standards for IoT application in the cellular networks [1]. In 5G standard, it is possible to allocate different subcarrier bandwidths to different groups of users for IoT application within a cell. In this paper, as in 5G standard, 5 different subcarrier bandwidths $W_i = 2^i \times 15kHz, i \in \{-2, \dots, 2\}$ for 5 groups of users in a cell for IoT network are considered [2], [3].

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In telecommunication industry and infrastructure, dimensioning and planning play crucial rule. As the telecommunication goes toward 4G and 5G, the problem of dimensioning and planning in the network has changed. It is due to the fact that the new type of the radio network brings substantial difference in the complexity and the number of dimensioning problems. The basic tasks of the 4G and 5G network remain the same; therefore, it leads to have basic algorithms and tools for analyzing and dimensioning of the wireless networks [22]. 5G New Radio use many techniques which are in common with LTE such as OFDM, so dimensioning in 5G network inherits the same principles from 4G networks. The aim of the dimensioning in 5G network is to find the number of the required radio resources to answer the Quality of Services (QoS) requirement at a target user. Due to the limited communication resources in 5G networks, the effective dimensioning is an important problem to improve the network performance, especially in the presence of a large number of devices connected to the internet. In this work, we investigate the problem of subcarrier dimensioning in OFDMA-based IoT network, including the number of available subcarriers, to satisfy the Signal to Interference Noise Ratio (SINR) requirements of all the devices in the system.

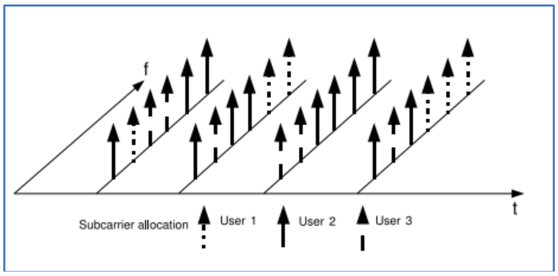
The randomness of users' location limits the performance of the IoT network. Location of users is becoming more and more random. To address this issue, stochastic geometry or random point processes gets importance in the wireless network. Point processes are used to model the position of base stations or users in a network. The model that we characterize in our system is the Poisson point process. We want to introduce an IoT framework for dimensioning 5G networks in order to theoretically and numerically analyze the performance of the system in downlink. This work is inspired from the works of Decreusefond, Laurent et al [4], [5].

Nowadays with increasing demand in high data rate services, Orthogonal Frequency Division Multiplexing (OFDM) has attracted much attention and has been selected as the based modulation technique for the 5G. OFDMA is a multiuser version of the OFDM digital modulation scheme. This allows simultaneous data rate transmission from several users.

Multiple access is achieved in both cellular networks (Long Term Evolution (LTE) mobile networks) and wireless LAN (802.11) by assigning subsets of subcarriers which are called subchannels to individual users.

The scheduling of radio resources would be easier by subchannels, but considering a subchannel as a resource unit requires the estimation of channel gain. This issue can be addressed by assuming that every subcarrier in each subchannel experiences the same channel gain [5]. The implementation issues such as the scheduler design and physical layer signaling that have been existed for assigning subcarriers to different users are removed. The contribution of 5G standards for the IoT networks is that we are able to allocate directly subcarriers to different users instead of subchannels. In this work, a subcarrier is considered as a resource unit for dimensioning the OFDMA- based network [2], [3]. The principle of OFDMA subcarrier allocation is presented in Figure 1 [4]. The number of the required subcarriers by each user depends on the channel gain and the needed bit rate of that user. In these systems, some packets would be lost due to the overloading which happens when the number of demanded subcarriers by the whole users in the cell is greater than the number of available subcarriers at the BS. The question we want to answer here is "How many subcarriers a BS should possess to ensure a small loss probability?". The answer to this question depends on having a good estimation of the loss probability.

In this paper, we propose an upper bound for the error probability of the loss communication. Our analysis is an approximation, but will be shown that provides reliable results. By using the proposed upper bound, it is then possible to calculate the number of required available subcarriers at the BS.



Principle of OFDMA sub-carrier allocation [4]..

First, a general model for a cell working on an OFDMA system in 5G network by considering an IoT application is proposed. We consider different groups of users with their unique bandwidth and capacity requirement. For the Poisson distribution of the users' locations, the distribution of the number of the required subcarriers by each user would be Poisson as well. In fact, for having the exact result, it is needed to have huge capacity for the memory. Therefore, we derive approximation formula by using concentration inequality for functions defined on the PPP which is efficient at a very low cost.

This paper is organized as follows. In Section 2, we describe the related works. In Section 3, the system model which include an introduction to Poisson point process and dimensioning

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problem is proposed. In Section 4, the numerical results for performance analysis of the system is presented and finally a brief conclusion of the paper is given in Section 5.

II. Related Works

Many works explore dimensioning and resource allocation in OFDMA-based networks. Resource allocation mostly comes with power or subcarriers allocation in an optimization problem, Agarwal et al [6] consider the dynamic allocation of power and rate on each tone in downlink OFDMA networks based on the knowledge of the channel conditions at the BS and propose two algorithms for solving the Weighted Sum Rate Maximization (WSRMax) problem, which finds the tone and power allocation that maximizes the weighted sum rate with the total power constraint, to reduce both considerable feedback-overhead and high computation complexity. Karray et al [7] consider the WSRMax problem and Weighted Sum Power Minimization (WSPMin), which is a dual problem of WSRMax, and employ the Lagrange dual decomposition method to solve both optimization problems. In [8], the authors consider the sum rate maximization problem in multi-user OFDM (mu-OFDM) and have proposed a suboptimal algorithm to the constrained fairness problem which assures each user can achieve a required data rate. In [9], the authors consider two resource allocation schemes for the BS: first, the BS does not have knowledge of its users' channels. Second, the BS takes the fading into account for the resource allocation problem and they compare the performance of these two schemes. Huang et al [10] use a gradient-based scheduling scheme [11] in order to reduce the resource allocation problem to an optimization problem which can be solved during each time-slot.

The OFDMA planning and dimensioning have been more recently under investigation which is what we investigate in our work, for instance, authors in [16] work on cell dimensioning for WiMAX which is an OFDMA-based network. Cell dimensioning is the first step toward radio network planning which aims to estimate the number of the BSs. In this case, parameters for theoretical and technical are taken from IEEE 802.11 standard. They introduce a dimensioning tool named DoORs. This tool enables cell designing faster and easier. In paper [19], authors does dimensioning for OFDMA networks in two steps. The first step is radio coverage and the goal of this step is to determine the modulation and coding scheme probabilities for a user from the spatial SINR distribution over the cell. This step requires excessive simulations which they propose a semi-analytical approach to reduce the simulation time. The second step is traffic analysis. Traffic analysis enables to acquire the dimensioning parameters such as the number of active users and the spectral efficiency per user. This analysis is based on Markovian approach. These two analysis are proposed for dimensioning OFDMA networks. Authors in paper [18] work on IoT network based on OFDMA such as NB-IoT and LTE-M. They assume that nodes in this wireless sensor network are distributed by spatial PPP. They propose a statistical method to model and analyze the behavior of this network which is used for resource dimensioning in the uplink communications in cellular IoT network with low transmission power and low delay requirements. Based on this, total number of the required radio resources are calculated. At last, they analyze the performance numerically and compare it with empirical simulation results. The results of the comparison shows the performance of the proposed model is close to the experimental results. [20] has taken a new approach and uses the congestion probability for radio resource dimensioning. In this approach, in a 5G cell, the position of the users in the indoor is modeled with spatial PPP and the distribution of users in the outdoor is modeled by a linear Poisson process. The total number of the requested radio resources has compound Poisson distribution. By using combinatorial analysis, especially the exponential Bell polynomials, the congestion probability is derived. The dimensioning of the radio resources is done by having the given value of the congestion probability and knowing the relation of the resources and area throughput in the cell. This approach is justified numerically. In [21], radio resource dimensioning with a statistical model for a cluster of a LTE mobile Ad Hoc network is studied. The objective of this work is to compare the effect of using different transmission modes and the bandwidth allocated to cluster head on serving the cluster nodes. In this work, the statistical behavior of this network is studied and it is assumed that the active nodes are distributed randomly by a PPP. The dimensioning mainly has been carried out with computing the upper bound of the outage probability of the resources. They numerically and using Monte Carlo simulation, verify the analytical model. Karray et al [12] consider the cell planning problem on OFDMA-based network. First, they drive a closed-form expression for the outage probability formula of this system and then the simplified planning procedure by Gaussian approximation for the effective SINR of the system is introduced. They consider Erlang's loss model and have evaluated the minimal density of BSs which gives an acceptable blocking probability for streaming traffic. In [4], the authors consider the exact expression of loss probability for an overloaded system in LTE systems and analyze it numerically. Then, present Edgeworth expansions for the loss probability, by that they show one can have guaranteed dimensioning. They propose

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concentration inequality for functions defined on the Poisson point processes which guarantee a loss probability is less than a given threshold. Taking this paper in to account, we give an analytical model for dimensioning in terms of subcarriers in IoT networks by considering 5G standards for that. We analyze the performance of the loss probability upper bound for having acceptable dimensioning. We do not present the approximate expression for the loss probability. In [5], the authors classify users in to different classes according to their traffic patterns in the LTE system and they model the position of users by Poisson point process. They consider path-loss and shadowing as large scale fading in the Shannon capacity of the system. With using concentration inequality, they drive the upper bound probability of loss communication for the system which leads to acceptable subchannel dimensioning in OFDMA-based networks. Inspired by this work, we propose an upper bound for the loss probability by using stochastic geometry and specifically Poisson point process in overloaded OFDMA-based 5G networks by considering IoT application.

We analyze the downlink performance of the subcarrier dimensioning in these networks by classifying users in to different groups with specific bandwidth of each group in 5G standard for IoT application and taking in to account the data rate requirement of each group.

SYSTEM MODEL

A. POISSON POINT PROCESS

The randomness of the users' locations leads us to use the Poisson point process (PPP) to model the position of the users by considering them as points on a mathematical space. See Appendix A, to review the existed theorems and definitions in the PPP such as the Superposition Theorem for getting the superposition of independent PPPs, Campbell Theorem to get the average and the variance of a function summed over a Poisson point process and the Concentration Inequality Theorem for getting the upper bound of the error probability in next section of my paper for dimensioning the subcarriers.

B. DIMENSIONING

Circular cell B is considered with radius R where 5 groups of active users are competing to have access to the available number of subcarriers in the base station (BS) of the cell. Note, having 5 groups is due to that there are 5 different subcarrier bandwidths. Pt is the transmit power by the BS associated with each user. The signal power received by user equipment at position x is $P_t K\bar{g}||x||^{-\gamma}$ where K is a constant separates the far field from near field propagation due to attenuation at a reference distance [5]. The path-loss exponent is γ, which determines how path-loss increases with distance. Its variations depend on the propagation environment, for example in an urban area, it changes from 3 to 5. In here for the sake of simplicity, it is considered as a constant. The mean value of the shadowing in our system is denoted by $\bar{g}$.

Each user within a group i has its corresponding subcarrier bandwidth Wi and capacity Ci. Considering path-loss and mean of the shadowing as large scale fading in our system model, the $SINR$ at user equipment is $P_t K\bar{g}(I+\text{n})^{-1}||x||^{-\gamma}$ where I is the inter-cell interference and n is the noise power. For the sake of simplicity, we assume that $I=0$ and minimum value of the $P_t K\bar{g}(\text{n})^{-1}||x||^{-\gamma}$ is the same for each group and is equal to β_{min} which means a user is able to receive the signal if the $SINR$ at his position is greater than β_{min}. A user is characterized by his position, and we model this as 5 independent Poisson point processes, $\phi_i, i \in \{-2,\dots,2\}$, on $B(0,R) = \{x \in R^2, ||x|| \le R\}$ with intensity measure $\Lambda_i(dx) = \lambda_i dx$ for group i. As a result of Theorem 2, the superposition is a Poisson point process, ϕ, with intensity $\Lambda(dx) = \sum_{i=-2}^{2} \Lambda_i(dx)$. The number of subcarriers demanded in the cell by a user at position is denoted by $N(x)$. By considering the capacity of this system, the number of the subcarriers for a user in the group i of the cell is,

$$N_i(x) = \begin{cases} \left\lceil \dfrac{C_i}{W_i.\log_2\left(1+\dfrac{P_t K\bar{g}}{n.||x||^{-\gamma}}\right)} \right\rceil & if\ \dfrac{P_t K\bar{g}}{n.||x||^{-\gamma}} \ge \beta_{min} \\ 0 & \end{cases}$$

where $W_i = 2^i \times 15kHz, i \in \{-2,\dots,2\}$. For each group of users, $N_{i,max} = \left\lceil \frac{C_i}{W_i.\log_2(1+\beta_{min})} \right\rceil$ is the maximum number of subcarriers required by a user in group i. The aim is to evaluate loss probability, which is:

$$P_{loss} = P(\int_B N\, d\phi \ge N_{av})$$

where N_{av} is the number of available subcarriers at the BS.

Theorem 1. with the assumptions of this section and by considering α as coefficient of the mean m,

$$P_{loss} = P(\int_B N(x) d\phi(x) \ge \alpha m) \le P_{up,b}(\alpha)$$

where

$$P_{up,b}(\alpha) = \exp\left(-\frac{v}{N_{max}^2} g\left(\frac{(\alpha-1)mN_{max}}{v}\right)\right)$$

with $N_{max} = max_{i\in\{-2,\dots,2\}}\{N_{i,max}\}$,

$$m = P(\int_B N(x)\Lambda(dx) = \int_B N(x)\lambda dx,$$

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and

$$v = P(\int_B N(x)^2 \Lambda(dx) = \int_B N(x)^2 \lambda dx,$$

Proof. Rewriting equation 3 as follow

$$P\left(\int_B N(x) d\phi(x) \geq \alpha m\right) = P\left(\int_B N(x) d\phi(x) - m \geq (\alpha - 1)m\right),$$

helps to use Theorem 4 for proving this theorem. By Definition 3, we have $(\forall i \in \{-2, \ldots, 2\}) \mid D_x N_i(w) \mid \leq N_{i,max}$, and for the whole system we have,

$$\max|D_x N_i(w)| = \max_{i\in\{-2,\ldots,2\}} |D_x N_i(w)|$$

then the N_{max}, used in Theorem 1, is $\max_{i\in\{-2,\ldots,2\}} \{Ni, max\}$ and the upper bound is as follows

$$P_{up,b}(\alpha) = \exp\left(-\frac{v}{N_{max}^2} g\left(\frac{(\alpha-1)mN_{max}}{v}\right)\right)$$

By taking $N_i(x)$ into account as a piece-wise constant and increasing with respect to x, both m and v. Can be computed. Considering $R_{i,j}, j = 1, 2, \ldots, N_{i,max}$ and $R_{i,0} = 0$, we have

$N_i(x) = j$ for $||x|| \in [R_{i,j-1}, R_{i,j})$. By having Equation 1, $R_{i,j}$ can be determined as following:

$$R_{i,j} = \left(\frac{P_t K \bar{g}}{n(2^{C_i/(j.W_i)} - 1)}\right)^{\frac{1}{\gamma}}$$

then

$$m = \int_B \mathrm{N}\, d\Lambda = \sum_{i=-2}^{i=2} \int_B \mathrm{N}_i\, \lambda_i dx$$

$$= \sum_{i=-2}^{i=2} \lambda_i \sum_{j=1}^{N_{i,max}} j \int_{2\pi} \int_{R_{i,j-1}\wedge \mathrm{R}}^{R_{i,j}\wedge \mathrm{R}} r dr. d\phi$$

$$= \pi \sum_{i=-2}^{i=2} \lambda_i \sum_{j=1}^{N_{i,max}} j(R_{i,j}^2 \wedge \mathrm{R}^2 - R_{i,j-1}^2 \wedge \mathrm{R}^2)$$

and

$$v = \int_B \mathrm{N}^2\, d\Lambda = \sum_{i=-2}^{i=2} \int_B \mathrm{N}_i^2\, \lambda_i dx$$

$$= \sum_{i=-2}^{i=2} \lambda_i \sum_{j=1}^{N_{i,max}} j^2 \int_{2\pi} \int_{R_{i,j-1}\wedge \mathrm{R}}^{R_{i,j}\wedge \mathrm{R}} r dr. d\phi$$

$$= \pi \sum_{i=-2}^{i=2} \lambda_i \sum_{j=1}^{N_{i,max}} j^2(R_{i,j}^2 \wedge \mathrm{R}^2 - R_{i,j-1}^2 \wedge \mathrm{R}^2)$$

where $a \wedge b = min(a, b)$. For finding the value of Nav which grantees that the loss probability is less than a given value, we can take $N_{av} = \alpha m$ in to account and by substituting α in terms of N_{av}, the upper bound of the loss probability in Theorem 1 is,

$$P_{up,b}(\alpha = N_{max}/m) = \exp\left(-\frac{v}{N_{max}^2} g\left(\frac{(N_{max}/m - 1)mN_{max}}{v}\right)\right)$$

III. Numerical Result

In this section, the loss probability upper bound in the mentioned system model for subcarrier dimensioning is considered. The capacity required by each user in group i, $(\forall i \in \{-2, \ldots, 2\}) C_i$, is fixed to $C_{-2} = \frac{4kb}{s}, C_{-1} = \frac{8kb}{s}, C_0 = 12kb/s, C_1 = 30kb/s$, and $C_2 = 48kb/s$ respectively, and the subcarrier bandwidth allocated to each user in group i is $W_{-2} = 3.75kHz, W_{-1} = 7.5kHz, W_0 = 15kHz, W_1 = 30kHz$, and $W_2 = 60kHz$ respectively. We also consider $PtK\bar{g}/n = 106$ and $\beta_{min} = 0.2$. The radius of the cell is $100m$. For the first six figures the users intensity of each group is the same $\lambda_{-2} = \lambda_{-1} = \lambda_0 = \lambda_1 = \lambda_2$. The users intensity of the system is $\lambda = \sum_{i=-2}^{2} \lambda_i$. Figures 2, 3, and 4 show $log10(Pup.b/Ploss)$, the logarithm of the ratio of the upper bound to the exact value of the loss probability, versus λ for three different ranges of λ for three path-loss values $\gamma = 3.4, 3.2$, and 3 and $\alpha = 1.5$. The first figure is for high intensity of users, the second one is for normal intensity of users, and the third one is for low intensity of users. We can see that the proposed upper bound works well and it is acceptable in the all three range of users intensity. In Figures 3, and 4, the logarithm of the ratio versus λ is linear. We can use this fact to verify the correctness of our simulation since from what we have from the derived formula for the upper bound of the error probability, it would be linear versus λ when the intensity of the users increases and passes the certain level. As it is shown in the figures, by increasing the intensity, the upper bound becomes closer to exact value. In Figures 3 and 4, the logarithm of the ratio versus λ is linear. We can use this fact to verify the correctness of our simulation since from Figures 5, 6, and 7 show $log10(Pup.b/Ploss)$ versus α for $\lambda = 0.005, \lambda = 0.002$, and $\lambda = 0.0002$ respectively, considering three path-loss values $\gamma = 3.4, 3.2, and\ 3$. We find out that by changing α in range $[1.2, 2]$ the proposed upper bound works well and it is acceptable in all three range of users' intensity. It is demonstrated in the figures that changing α in each given range of λ does not have the same behavior. Its changes have more effect on the ratio in normal and high users' intensity but is negligible in low users' intensity. Figures 8, 9 show the ratio versus one of the groups' intensity, $\lambda 2$, for constant users intensity of the system $\lambda = 0.002$ and $\lambda = 0.004$ respectively,

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when $\lambda_{-2} = \lambda_{-1} = \lambda_0 = \lambda_1$. It can be concluded from the figures that for the constant user intensity in the system, changing one of the groups' intensity has a negligible effect on the ratio, and the proposed upper bound is still acceptable. Another parameter that we can check its effect in our system is path-loss.

As shown in Figures 4, 7 changing path-loss has negligible effect in low users' intensity, but it is obvious from the figures related to normal and high users' intensity that for the higher values of path-loss in these systems, the upper band is closer to the exact value of the loss probability.

The advantage of using the proposed upper bound instead of computing the exact value is in the memory usage. Numerically, we can see that the memory needed for computing the exact value is 10^8 bigger than the memory needed for computing the upper bound.

The results show that the proposed upper bound of the loss probability in the considered system model in 5G networks works well and is acceptable. It can be used in dimensioning of the OFDMA subcarriers in IoT application for finding the value of N_{av}, the number of available subcarriers at BS, which guarantees that the loss probability is less than a given probability.

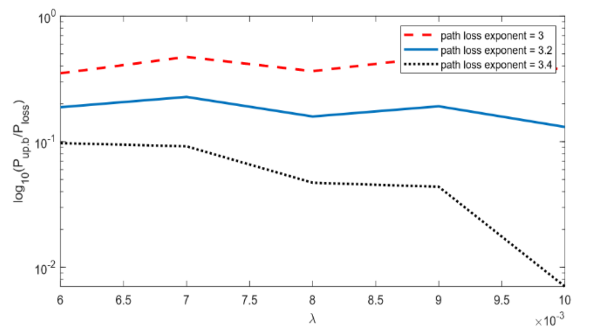

Figure 1. Ratio of the upper bound to the exact value of the loss probability versus system intensity λ for the mean coefficient $\alpha = 1.5$.

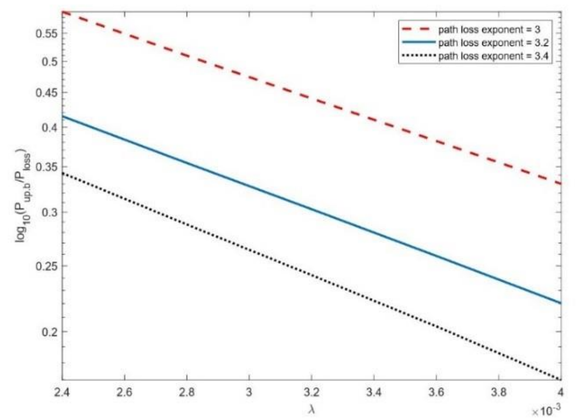

Figure 2. Ratio of the upper bound to the exact value of the loss probability versus system intensity λ for the mean coefficient $\alpha = 1.5$.

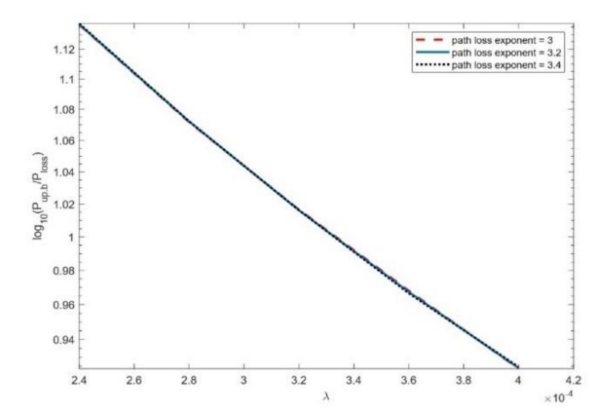

Figure 3. Ratio of the upper bound to the exact value of the loss probability versus system intensity λ for the mean coefficient α = 1.5.

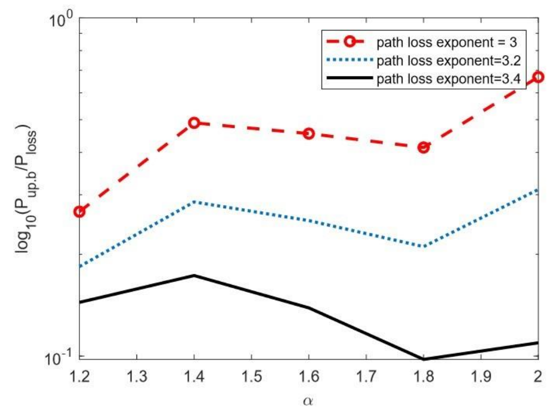

Figure 4. Ratio of the upper bound to the exact value of the loss probability versus the mean coefficient α for system intensity λ = 0.005.

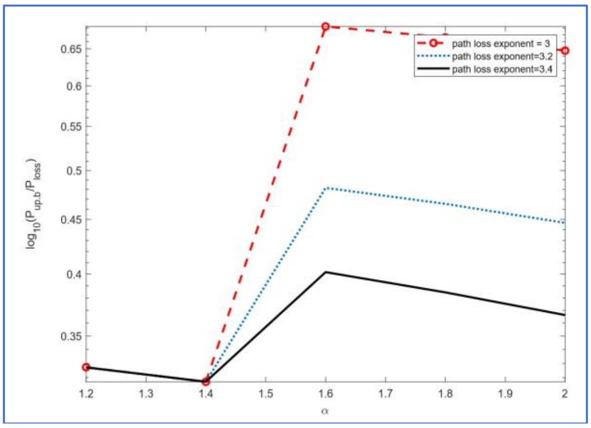

Figure 5. Ratio of the upper bound to the exact value of the loss probability versus the mean coefficient α for system intensity $\lambda = 0.002$.

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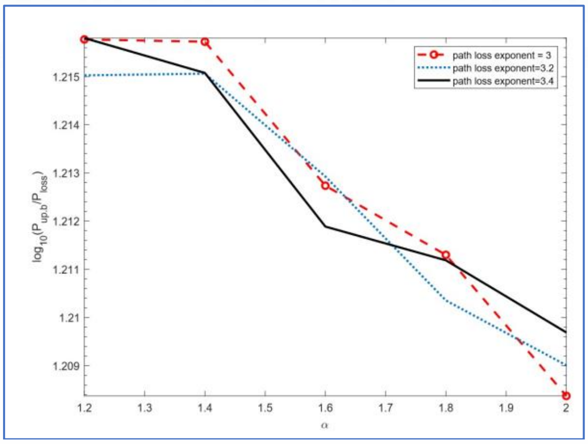

Figure 6. Ratio of the upper bound to the exact value of the loss probability versus the mean coefficient α for system intensity $\lambda = 0.0002$.

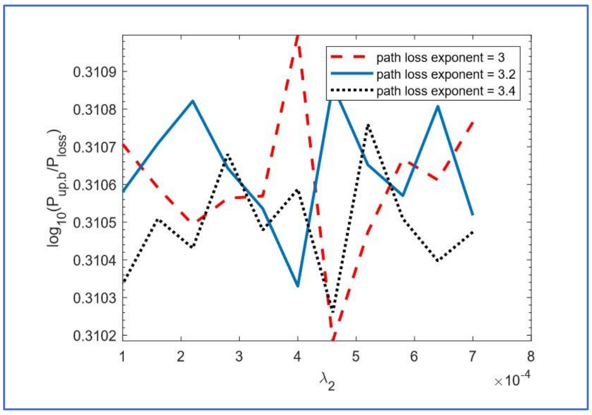

Figure 7. Ratio of the upper bound to the exact value of the loss probability versus intensity of group 2, λ_2, for the mean coefficient $\alpha = 1.5$ and system intensity $\lambda = 0.002$.

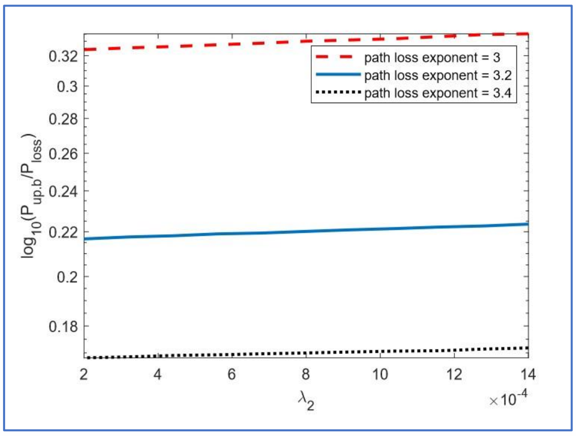

Figure 8. RPL Ratio of the upper bound to the exact value of the loss probability versus intensity of group 2, λ_2, for the mean coefficient $\alpha = 1.5$ and system intensity $\lambda = 0.004$.

IV. Conclusion

In this paper, the dimensioning of subcarriers in downlink by using stochastic geometry in IoT framework of 5G networks is presented. The concentration inequality on Poisson space for getting the upper bound of the loss probability is used, which happens when the number of demanded subcarriers is greater than the number of available subcarriers. It is proved that we can compute the upper bound of the loss probability in order to have subcarrier dimensioning in the IoT networks.

The performance of the proposed upper bound system in the different range of users' intensity and different path-loss exponent is numerically investigated. It is shown that the proposed upper bound is acceptable in each case and has a substantial less numerical complexity.

Appendix A. Review of PPP

In this section, I elaborate on the basic concepts of the Poisson point processes [13], [14], [15] in more details. It is defined by the following definitions. We denote $N(A)$ the number of points of a point process ϕ falling in the Borel set A [13]. If some points form the Poisson point process, the number of the points would be a random variable with Poisson distribution.

Definition 1. *Let $\Lambda(.)$ be a Borel measure (a measure defined on the Borel sets) on K, then a point process ϕ on K is a* Poisson point process with intensity $\Lambda(.)$ if:

1) *$N(A)$ is poisson distributed with mean $\Lambda(A)$ for every bounded Borel set A included K,*

$$\{N(A) = k\} = e - \Lambda(A)\Lambda(A)k/k!$$

2) *For any k disjoint bounded Borel sets $A_1, \dots, A_k$ the random variables $N(A_1), \dots, N(A_k)$ are independent.*

It is noteworthy to mention that for the Poisson point processes, the intensity measure and the mean measure coincide.

Definition 2. *A configuration η in R^k is a set $\{x_n, n \geq 1\}$ where for each $n \geq 1, x_n \in R^k$ and $x_n \neq x_m$, for $n \neq m$. We denote by γ_{R^k} the set of configuration in R^k. A point process ϕ is a random variable with values in γ_{R^k}, i.e., $\phi(w) = \{x_n(w), n \geq 1\} \in \gamma_{R^k}$ [4].*

Here, we present the main theorems of a Poisson point process (PPP) which are needed for the dimensioning of sub-carriers in OFDMA systems in our work, such as the Theorem 2. (Superposition Theorem) which is about the superposition of independent PPPs, Theorem 3. (Campbell Theorem) giving the average and the variance of a function defined on a PPP and the Theorem 4. (Concentration Inequality Theorem) giving the upper bound of the error probability.

Theorem 2. *(Superposition [15]) For n independent Poisson point processes $\phi_1, \dots, \phi_n$ of intensities $\Lambda_1, \dots, \Lambda_n$ with $n < \infty$, the superposition $\phi = \cup_{i=1}^{n} \phi_i$ is known to be PPP with intensity $\Lambda = \sum_{i=1}^{n} \Lambda_i$.*

Theorem 3. *(Campbell Theorem [15]) Let ϕ be a*

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PPP of intensity function $\lambda(x)$, *i.e.*, $\Lambda(dx) = \lambda(x)dx$ *and* $f(x): R^k \rightarrow R^{+}$*and:*

$$F(w) = \int_{R^k} f(x)d\phi(x) = \sum_{n\geq 1} f(X_n(w)),$$

then

$$m_F = E\{F\} = \int_{R^k} f(x)\Lambda(dx) = \int_{R^k} f(x)\lambda(x)dx,$$

and

$$v_F = Var(F) = \int_{R^k} |f(x)|^2 \Lambda(dx).$$

Definition 3. *For* $F: \gamma_{R^k} \rightarrow R$ *and* $x \in R^k$, *we define*

$$D_x F(w) = F(w \cup \{x\}) - F(w),$$

note for $F = f d\varphi$, *we have* $D_x F = f(x)$.

Theorem 4. *(Concentration inequality [15]) Assume that* ϕ *is a Poisson point process on* R^k *with intensity* λ. *Considering* F, m_F, *and* v_F *are defined in the Theorem 3 and let* s *be the bound of* $|D_x F(w)| \leq s$. *Then, for any* $t \in R$,

$$P(F - m_F \geq t) \leq \exp\left(\frac{v_F}{s^2} g\left(\frac{t_s}{v_F}\right)\right)$$

where $g(x) = (1 + x)\ \ln(1 + x) - x$.

Further works can be done to increase the bandwidth of the proposed high gain antenna.

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