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GSSC: Graph Summarization based on both Structure and Concepts

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Abstract— **In this paper, we propose a new method for graph summarization named GSSC, Graph Summarization based on both Structure and Concepts. In this method, an attributed graph is summarized by considering both of its topology and related concepts. In this method, for a given attributed graph a new graph is constructed that an edge in this new graph represents structural and conceptual similarity of its two end points. Structural and conceptual similarity of two nodes not necessarily has the equal amount of importance in the weight of the resulting edge. For example, for a special case such as query answering, structure can be more important and vice versa. Similarity of two nodes is computed based on Jaccard similarity. This method has some advantages such as flexibility, simplicity, learning capability, user-orientation that makes it a better method for graph summarization. We implemented our method and the method proposed by Bei and evaluated these two methods on real-life dataset HEP_TH. Our experimental results showed effectiveness and efficiency of our proposed method.**

***Keywords-* Graph summarization; super-node; similarity; conceptual summarization; summary.**

I. INTRODUCTION

Graphs are used in a variety of applications for modelling data and their relationships. Examples of data modelled by graphs include social networks, communication networks, web graphs, biological networks, chemical compounds, etc. Graph theory and its applications has attracted the attention of the scientists [1] and specially there is a survey of existing work on graph matching, describing variations among problems, general and specific solution approaches, evaluation techniques, and directions for further research [2]. In that survey an emphasis is given to techniques that apply to general graphs with semantic characteristics.

These days many applications generate large scale and massive graphs with billions of nodes and edges and a lot of research has been done on theory and engineering of Tera-scale graphs [3]. In fact, we are faced with graphs which are very massive and their growth rate is also increasing rapidly. For example Facebook has had 1.11 billion members on March 2013 while at the end of 2004 had only about 1 million members (http://news.yahoo.com/number-active-users-facebook-over-230449748.html).

Query answering on these massive graphs is very time-consuming. Graph summarization has proposed as a solution for this problem. Recently several graph summarization algorithms [4], [5], [6], [7] have been proposed that reduce a massive graph to a smaller one by removing its details but preserving its overall properties. This smaller graph can then be used for query answering. Of-course these answers are not exact and have a little error. This error is acceptable because of lower response time which is a necessity in a lot of applications.

The formal definition of a summary graph according to [7] is as follows:

Definition 1. (Summary Graph) Let $G = (V, E)$ and $\Phi = \{V_1, V_2, V_3, \dots, V_k\}$ is a partition of G such that $\bigcup_{i=1}^{k} V_i = V$ and $\forall i \neq j: V_i \cap V_j = \emptyset$. The summary of G based on Φ is $S = (V_s, E_s)$ where $V_s = \Phi$ and $E_s = \{(V_i, V_j) | \exists u \in V_i \wedge v \in V_j \wedge (u, v) \in E\}$.

Fig 1 shows an undirected graph with 8 nodes and 13 edges and one of its summaries. In fact, a graph is partitioned into some parts each containing some nodes of the original graph. Each part is called a super-node. For example, as is shown in the Fig 1.a, the vertices a, b and c in the original graph are grouped together and make a super-node (blue one) in the summary graph (Fig 1.b). The edges between 2 super-nodes are also grouped together and shown by an edge in the summarized graph called a super-edge. For example six edges (a,d), (a,e), (b,d), (b,e), (c,d) and (c,e) are packed and shown by a super-edge between two blue and red super-nodes in the summary.

The summary graph has four super-nodes according to four dashed ovals in the original graphs as shown in Fig 1.b. To illustrate more, the super-nodes have the same color as their corresponding groups in the original graph.

A super-edge in the summarized graph shows that an edge must exist in the original graph between a node of the first super-node and another node from the second super-node. For example the super-edge between the red and blue super-nodes shows at least one of the edges (a,d), (a,e), (b,d), (b,e), (c,d) and (c,e) must exist in the original graph. The super-edge between the red and yellow super-nodes indicates that one of the edges (d,f) and (e,f) must exist in the original graph. Here the edge (e,f) exists in the original graph.

The formal definition of a good summary according to [7] is as follows:

Definition 2. (Good Summary) Let set of nodes in super-node V_i participate in the relationship (V_i, V_j) is as $P_{i,j} = \{u | u \in V_i \wedge \exists v \in V_j \text{ such that } (u, v) \in E\}$. The participation ratio of the relationship (V_i, V_j) is defined as $r_{i,j} = (|P_{i,j}| + |P_{j,i}|)/(|V_i| + |V_j|)$.

For a group relationship, if its participation ratio is greater than 50%, then we called it a strong group relationship; otherwise, we call it a weak group relationship. Note that in an **ideal** summary, the participation ratios are either 100% or 0%. The participation ratios of a good summary are near to either 100% or 0%.

In fact a good summary of a big graph is the one that can be stored in the memory and the more important it should generate answers to the queries the same as answers generated from the original graph. This is the main challenge of the graph summarization.

The majority of summarization algorithms generate structural summaries while most real world graphs are attributed graphs in which every node or edge has a lot of attributes. In this kind of graphs, node attributes are important and must be considered in summarization.

Usually users are interested in summarization based on concepts (attributes). For a given attributed graph, a lot of summaries can be produced according to selected attributes.

The formal definition of an attributed graph is as follows:

Definition 3. (Attributed Graph) An attributed graph is defined as 4-tuple $G = (V, E, \Sigma, F)$ where $V = \{v_1, v_2, \dots, v_n\}$ is a set of n nodes, $E = \{(v_i, v_j) | 1 \le i, j \le n \text{ and } i \neq j\}$ is a set of m edges, $\Sigma = \{a_1, a_2, \dots, a_L\}$ is a set of L attributes. Attributes of a node $v_i \in V$ is denoted as $[a_1(v_i), a_2(v_i), \dots, a_L(v_i)]$ where $a_j(v_i)$ is an observation value of v_i on attribute a_j. The set $F = \{f_1, f_2, \dots, f_L\}$ denotes a set of L functions and each $f_i: V \mapsto dom(a_i)$ assigns each node $v_j \in V$ an attribute value in the domain $dom(a_i)$ of the attribute a_i $(1 \le i \le L)$.

The formal definition of hybrid summarization (summarization based on both structure and attribute similarities) is as follows:

Definition 4. (Hybrid Summary) For a given graph $G = (V, E)$ let:

1) Every node has attribute set $A_v = \{a_1, a_2, \dots, a_t\}$.
2) $\Phi = \{V_1, V_2, \dots, V_k\}$ is a partition on V.
3) User is interested in attributes $A_u = \{a_{i_1}, a_{i_2}, \dots, a_{i_j}\}$ where $A_u \subseteq A_v$.

Then a hybrid summarization is $G_S = (V_s, E_s)$ where:

1) G_S is a structural summary as previous.
2) All vertices inside V_i have the same value for all attributes in A_u.
3) The density of edges inside each V_i is more than a given threshold.
4) The edge density of edges between super-nodes V_i and V_j is less than a given threshold.

For some applications, conceptual summarization is necessary for analysing massive graphs, but in general a summary resulted from both structure and concepts may be useful. We propose a method for this purpose.

There is a method [8] for graph summarization based on both structure and concepts. This method unlike our method, at first summarizes a graph based on concepts or similarity of nodes and then tries to adjust the summary with the graph structure. In fact contribution of graph structure to summary is added after construction of conceptual summary. Zhou et al. [9] proposed an algorithm for graph clustering based on both structure and node similarity. In this method unlike our method similarity of two nodes is measured based on number of random walks between two nodes.

Henceforth our target is to have a method for graph summarization that generates structural or conceptual summaries or even a mixture of these two summaries

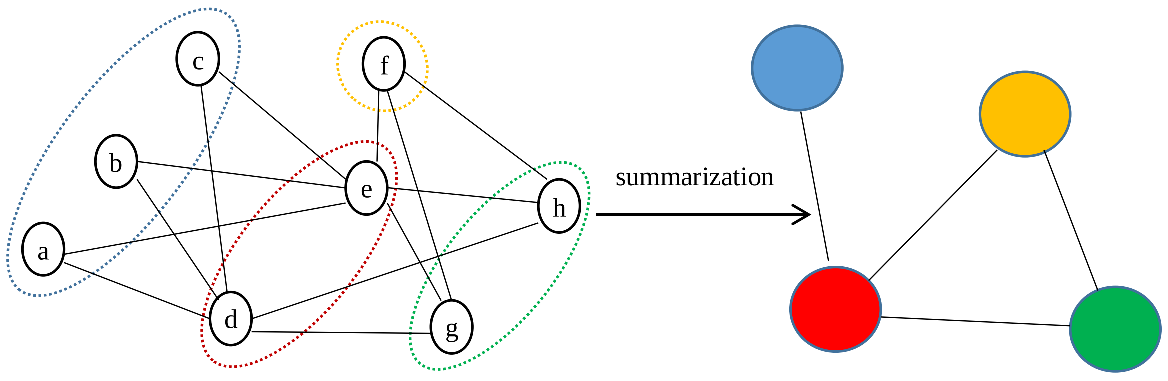


Fig 1: (a) The original graph (b) The summarized version of the original graph

with the arbitrary degrees of contributions. Such a method can be very useful in query answering based on learning degree of contributions of structure and attribute similarities in resulting summary. The more summary is realistic the more queries can be answered precisely. By learning the value of α (structure contribution in summary) based on trained dataset it can produce a realistic summary.

Motivating Applications: Graph summarization problem can be motivated by revealing biological modules [10], provenance systems [11] and many other applications. In the following, we further discuss these two applications.

Finding biologically meaningful modules in a network of proteins is important. In fact detection of protein complexes and prediction of biological processes can discover the global organization of the cell. Graph summarization can be used for this purpose.

Provenance systems produce provenance graphs that can be used for tasks such as determining the inputs to a particular process for debugging entire workfellow executions. Visualization can be used to support such tasks. By summarization, it is possible to visualize such massive graphs.

The rest of the paper is organized as follows. Related works is reviewed in Section 2. In Section 3 the new proposed method for graph summarization is presented. Evaluation and Experimental results are given in Section 4. Section 5 is dedicated to discussion and finally we concluded this paper in Section 6.

II. RELATED WORKS

In the following, some of recently proposed summarization algorithms are described shortly to illustrate the scope of the problem.

Navlakha et al. [5] proposed a summarization algorithm in 2008 where graph compression is done by collapsing a set of similar nodes into super-nodes and defining a supper-edge between every pair of super-nodes. It tries to construct a compression graph with the minimum representation cost based on the MDL[1] idea.

For this purpose, they developed two iterative algorithms, GREEDY and RANDOMIZED. The GREEDY algorithm selects, in each stage, the best pair of nodes to merge based on the representation cost reduction. It is obvious that the running time of this algorithm is high. To reduce the running time, a RANDOM- IZED algorithm has been proposed by the authors (Navlakha et al.). Unlike the GREEDY, in this algorithm two merging nodes are selected randomly.

In 2008, Tian et al. [6] proposed a summarization method with two summarization operations called SNAP2 and k-SNAP for grouping nodes and constructing summary. This summarization algorithm has been proposed for attributed graphs. Tian et al. defined attribute compatible grouping and also relation compatible grouping. In addition, they improved the SNAP operation by proposing k-SNAP, where k is the right size of resulting summary and is given by the user.

In 2009, Zhang et al. [7] have improved the k-SNAP operation in two ways. In fact, k-SNAP method has two shortcomings. First, users have to categorize the attribute values and second there is no criterion to measure the quality of the resulting summary. For these shortcomings, Zhang has proposed the CANAL algorithm to categorize attribute values automatically and a criterion to estimate the quality of the summary.

In 2008, Chen et al. [12] proposed the OLAP framework which provides OLAP like operations on graphs. The OLAP framework has been introduced to create cubes from graphs based on dimensions and measures. The natural property of OLAP framework is that constructs a summary based on the selected attributes and given input information.

Another summarization method has been proposed by Chen et al. [13] in 2009 for mining frequent patterns. This method works by producing randomized summary graphs. In fact, Chen et al. confirmed, in the case of massive graphs, that the traditional pattern mining algorithms are very time-consuming and inefficient because of random access time. Therefore, they proposed a summarization method that first constructs summaries and then mines them instead of mining original disk-resident graphs.

In [14] a method has been proposed for graph summarization that guarantees the quality of the summary. This method produces a summary that

[1] Minimum Description Length

minimizes the reconstruction error. The error is computed based on the difference between the adjacency matrices of the original and summary graphs. The authors have presented a connection between graph summarization and geometric clustering. Based on this connection, they have developed a polynomial-time algorithm to compute the best possible summary with a given size.

Navlakha's algorithm which was described as the first algorithm in this section is a bottom-up algorithm. The top-down approach starts with a minimum number of super-nodes and then iteratively splits super-nodes to achieve a summary with a right number of super-nodes. In this approach, a criterion is necessary to divide a super-node. In [6] there is an algorithm (Algorithm 2) to summarize graph based on top-down approach.

These days the majority of real applied graphs are attributed graphs such as social networks and web graphs. Recently a lot of papers have been published on attributed graphs. For example we can list papers for matching patterns [15], matching graphs with fuzzy attributes [16] and predicting links and inferring attributes on a social attribute network [17].

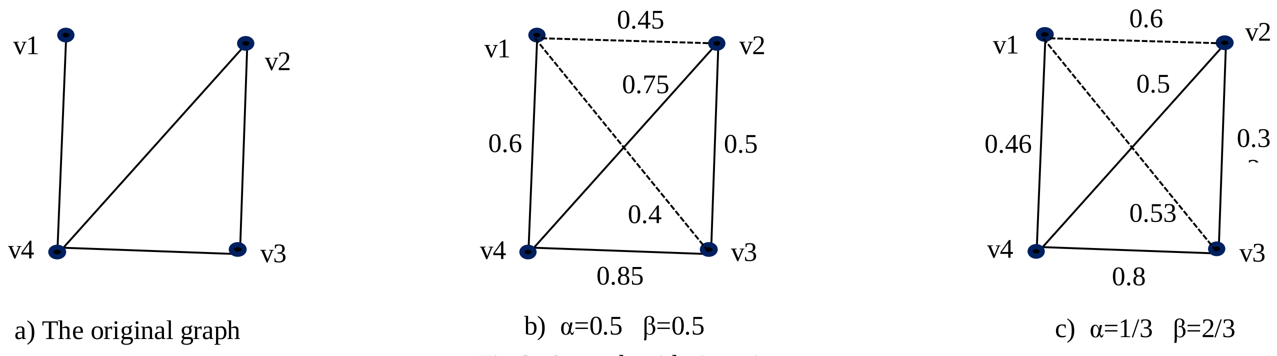


Fig 2: A graph with 4 vertices

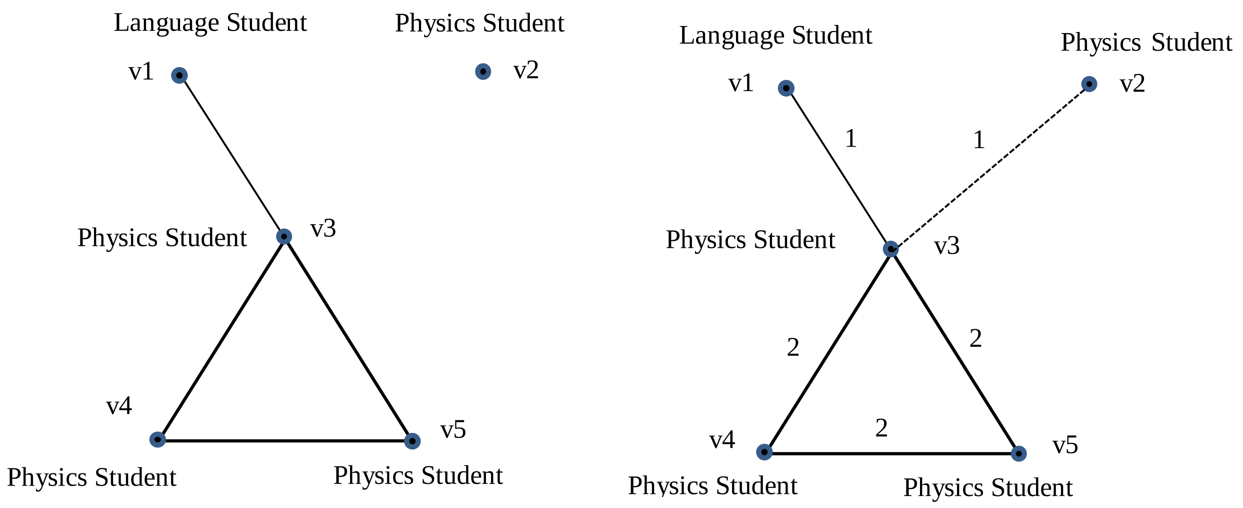


Fig 3: a) Student's graph b) New graph constructed based on proposed method

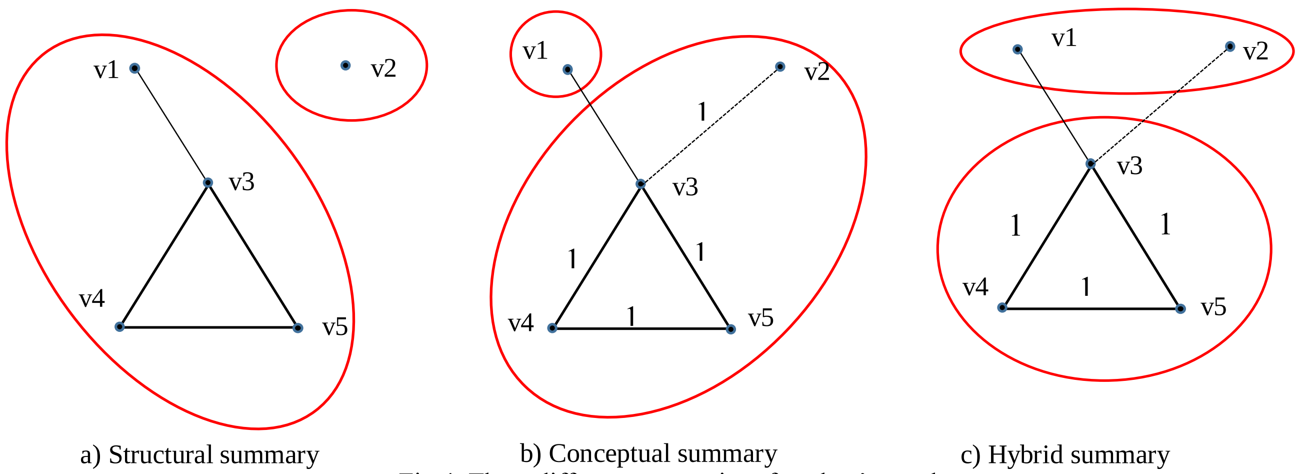
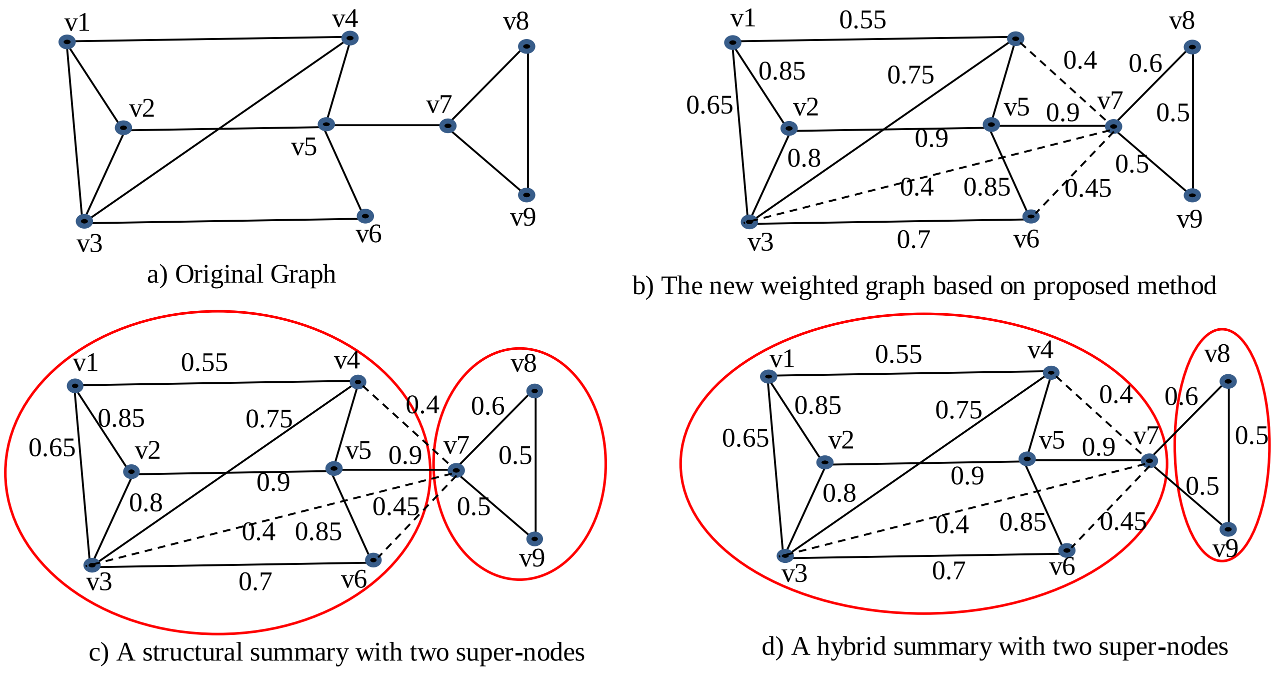


Fig 4: Three different summaries of student's graph


a) Original Graph

b) The new weighted graph based on proposed method

c) A structural summary with two super-nodes

d) A hybrid summary with two super-nodes

Fig5: An instance graph and its summaries

Therefore at present, hybrid summarization is more important than structural summarization. All above mentioned summarization methods are single-process solutions and as a result cannot scale to large graphs. In [18] three distributed graph summarization algorithms have been proposed.

Dynamic graphs and their interpretations are also important and in [19] these graphs have been studied, formulated and a new method is proposed for finding coherent and temporal patterns in these graphs.

Summarizing a graph based on both structure and attributes is important and a method [8] has been proposed for this purpose. The method summarizes a graph by introducing real and virtual links. In fact for a given graph, a new graph is constructed where the weight of an edge in this new graph is resulted from both real and virtual links.

III. PROPOSED SUMMARIZATION METHOD

The new proposed summarization method covers two above mentioned kinds of summarization, structural and attribute-based. In fact both structure and concepts (attributes) of the graph have contributions in making the resulting summary. For this reason, we consider two kinds of edges namely structural and conceptual edges. A structural edge is as previous and indicates that two vertices are connected, while a conceptual edge shows the similarity of two vertices based on their attribute values. In this new proposed method, for a given graph a new graph with the same vertices but with new edges is constructed. In the constructed graph, edges are weighted and some edges may be added because of attribute similarities of vertices. Weight of an edge is summation of structural and conceptual weights, of course weighting factors may be different. For more demonstration, let consider an attributed graph in Fig 2.a with similarity of vertices as given in Table 1.

Table 1: Vertices similarity of graph in Fig 2.a

Source	1	1	1	2	2	3
Destination	2	3	4	3	4	4
Similarity	0.9	0.8	0.2	0.0	0.5	0.7

Based on the given graph, two new graphs constructed. In the first new constructed graph, structural and conceptual similarity has equal contributions in the weight of an edge ($\frac{1}{2}$ and $\frac{1}{2}$). In the second constructed graph, the contribution of conceptual similarity is two times as structural similarity ($\frac{2}{3}$ $versus$ $\frac{1}{3}$).

For more illustration, let consider another example as shown in Fig 3.a. Graph in Fig 3.a is the original graph and its new constructed graph is shown in Fig 3.b. In the new graph, the edge (v_2, v_3) has added because of similarity of v_2 and v_3, similarity of these two nodes based on their attributes. Both of them study physics. In this case, contributions of structural and conceptual weights have considered equal.

Structural and conceptual summaries with two super-nodes for student's graph shown in Fig 4.a and 4.b respectively. Based on the new proposed method, student's graph can be summarized as shown in Fig 4.c.

As seen in Fig 4.c, hybrid summary (constructed based on GSSC) can be different with structural and conceptual summaries. In some cases this summary can be more reasonable and useful. As another example, consider the graph in Fig 5.a and let assume weights of edges are as shown in Fig 5.b. The structural summary of this graph shown in Fig 5.c and its summary based on GSSC method shown in Fig 5.d.

Up to now the new proposed method has been described informally and in the following section we tried to demonstrate this method formally. In this text, we will use conceptual and attribute-based summaries interchangeably.

A. Notations

In this section for more readability, the most frequently used symbols and abbreviations in this paper are described. We list symbols and their descriptions in the Table 2.

B. Proposed Method Computations

Weight of edges has an important role in summarization and for this reason the way these weights are calculated must be explained clearly. Weight of an edge is calculated as follows:

$$w(e) = \alpha w_{st}(e) + (1-\alpha) w_{si}(e) \quad (1)$$

$$0 \leq \alpha \leq 1$$

Where α and (1- α) are weighting of structural and conceptual similarity and show their contributions in the weight of an edge in new constructed graph. For equal contributions they are considered 0.5. For structural summarization, it is sufficient to consider α equal to one and for conceptual summarization α must be considered zero. The value of α depends on application and the aim of summarization.

The weight of w_{st} is as follows:

$$w_{st}(e_{i,j}) = \begin{cases} 0 & \mathrm{W[i][j]} = 0 \\ 1 & \mathrm{W[i][j]} = 1 \end{cases} \quad (2)$$

Where **W** is the adjacency matrix of the input graph.

Attribute-based similarity of two vertices is also a reason for overall similarity of two vertices. Attributes can be single or multi-valued. Similarity of two vertices based on the given attribute set $\{a_1, a_2, \dots, a_k\}$ with the importance degrees $\{c_1, c_2, \dots, c_k\}$ is calculated as follows:

$$w_{si}(e_{ij}) = \sum_{l=1}^{k} c_l w_{si}(e_{ij}, l) \quad (3)$$

In fact $w_{si}(e_{ij})$ is the similarity of two vertices v_i and v_j based on the given attribute set. The c_i parameter is the contribution or importance of attribute a_i in similarity of two vertices and has the following conditions:

1) $0 \leq c_i \leq 1$

2) $\sum_{i=1}^{k} c_i = 1$

Table 2: Symbols and abbreviations

Notation	Description
G	graph
s_i	ith super-node
den	Density
$e_{i,j}$	*edge between two vertices* v_i *and* v_j
$se_{i,j}$	super-edge between two super-edges v_i *and* v_j
$\#e_{p,q}$	# of edges between two super-nodes s_p and s_q
$c(a_i)$	contributions of the ith attribute
$w(e)$	*weight of edge* ***e***
$a_i(v_p)$	value of ith attribute on node v_p
$n_v(G)$	# of vertices of G
$n_e(G)$	# of edges of G
$w(e_{ij})$	weight of edge between two vertices v_i *and* v_j
$w_{st}(e_{i,j})$	structural weight of edge between two vertices v_i *and* v_j
$w_{si}(e_{i,j})$	attribute similarity weight of edge $e_{i,j}$
$w_{si}(e_{ij}, l)$	Similarity of two vertices v_i and v_j based on l th attribute.
$val(v_i, k)$	value of kth single-valued attribute of vertex v_i
$vals(v_i, k)$	values of kth multi-valued attribute of vertex v_i

For calculating similarity of two vertices based on a given attribute, we use the Formula 4. In fact depending on being single or multi-valued attribute, its calculation differs. The single-valued attributes are compared exactly while for multi-valued attributes, Jaccard similarity measure is used.

$$w_{si}(e_{ij}, k) = \begin{cases} 0 & a_k is\ single_{valued} \wedge val(v_i, k)! = val(v_i, k) \\ 1 & a_k is\ single_{valued} \wedge val(v_i, k) = val(v_i, k) \\ \dfrac{|vals(v_i,k) \cap vals(v_j,k)|}{|vals(v_i,k) \cup vals(v_j,k)|} & a_k\ is\ multi_{valued} \end{cases} \quad (4)$$

C. GSSC Algorithm

After constructing the new weighted graph, the graph can be summarized in top-down approach. In every step, edges with weight less than a given threshold are removed and graph is partitioned into some subgraphs. This trend continues to achieve a summary with the right size. We have presented this approach in algorithm 1.

D. Super-edge Weight Computation

The weight of a super-edge is computed based on weight of edges between nodes in two super-nodes. For two super-nodes with m and n nodes the weight of super-edge between these two super-nodes can be computed as follows:

$$w(se_{i,j}) = \frac{1}{m*n}\sum_{i=1}^{m}\sum_{j=1}^{n} w(e_{i,j}) \qquad (5)$$

Where $w(e_{i,j})$ is the weight of the edge between two super-nodes S_i and S_j. Edge weight is summation of structural and conceptual weights which have contributions α and $(1-\alpha)$ respectively. There are some distance/similarity measures such as cosine, n-norm, Jaccard, etc. that can be used for this purpose. Based on application and the aim of summarization, one of these similarity measures can be used. In some situations these similarity measures can be customized. The output of comparing two nodes is a number in interval [0 ... 1].

E. TIME COMPLEXITY

In the proposed method at first the weight of edge between every two vertices is calculated and after that the summary is generated by removing edges from less weighted toward high weighted. Thus the time complexity of the proposed algorithm is $O(|E| \times |V| \times |V|)$. This time complexity is for the worst case and in the best case when a large number of edges are removed in each iteration of algorithm, the time complexity is $O(|V| \times |V|)$.

IV. Evaluation and Experimental Results

To evaluate the proposed method, we selected a real-life HEP-TH dataset and implemented our method and SGVR method and run them on a system with configurations given in Table 6. The details of dataset, application and system are demonstrated in following subsections.

We compared our proposed method with the recently published paper on this subject which is Bei's method (SGVR). Authors of SGVR method have compared their method with other methods only based on density. Therefore for a fair comparison, we compared our proposed method with Bei's method based on density. Of course for comparing methods that summarize a graph based on both structure and concepts it is reasonable to compare summaries by considering both density and entropy.

A. Dataset

We considered real-life dataset HEP-TH, which presents information on papers in high-energy physics, for evaluation our proposed method. This dataset is an attributed graph which can be downloaded from knowledge discovery laboratory . Every vertex of this graph is one entity of type paper, journal, author or email_domain. The vertices are connected by attributed edges. The more information about this graph is given in the Table 3.

The number of each entity given in Table 4 and the number edges between every pair of these four entities given in Table 5.

Value of $\#e_{x,y}$ shows the number of edges between two entities representing by x and y. In fact sub-scripts x and y represent two first letters of four entities paper, author, journal and email_domain. For example pa, au, jo and em show paper, author, journal and email-domain respectively.

For summarization purposes, we considered a subgraph of this graph that only contains nodes of **paper** type. This subgraph has 29555 nodes.

```
----------------------------------------------------------------------------------------------------
Algorithm 1 Summarization (G, k , A, $\alpha$, C)
Input:  G: graph, k: the right size of the summary, A: user interested
         attributes, $\alpha$: the contribution of structure in the resulting summary,
        C: importance degrees of attributes
Output:  S: the resulting summary
   1.  Calculate the weight of edge between every two vertices as $E'$.
   2.  Construct a weighted graph $G' = (V, E')$ where $E' = \{(v_i, v_j) | v_i \text{ and } v_j \in V\}$
       based on $G = (V, E)$;
   3.  initialize $w_t$, $size$ and $\Delta w$;
   4.  num = $|\{G' | \text{where } G' \text{ is a connected copmponent of G and } |G'| > size\}|$
   5.  while $(num < k)$ {
   6.      $E' = E' - \{e_{ij} | w(e_{ij}) < w_t\}$
   7.      $w_t = w_t + \Delta w$;
   8.      $recalculate\ num, w_{deg}\ and\ w_{path}$
   9.    }
   10. Select the $k$ biggest connected components as super-nodes and make the summary graph
----------------------------------------------------------------------------------------------------
```

Table 3: The dataset information

#vertices	#edges	#vertex attributes	# edge attributes
42319	532430	39	15

Table 4: Number of entities in the dataset

#paper	#author	#journal	#email_domain
29555	9200	448	3116

Table 5: Number of edges between every pair of entities

$\#e_{pa,au}$	$\#e_{pa,jo}$	$\#e_{pa,Em}$	$\#e_{au,jo}$	$\#e_{au,Em}$	$\#e_{jo,Em}$	$\#e_{pa,pa}$	$\#e_{au,au}$	$\#e_{jo,jo}$	$\#e_{Em,Em}$
58515	20826	0	0	12487	0	352807	87794	0	0

For each paper, we considered three attributes num_revisions, downloads_60days and area. The number of nodes that has each of these attributes are 29555, 1566 and 3199 respectively. Two first ones are numerical attributes and the third one is an alphabetical field. Conceptual similarity of two nodes is measured based on these three attributes.

B. *Similarity Measures*

The similarity of two nodes can be measured based on their attribute values. Here we use exact comparison of corresponding fields to compute similarity of two nodes. In fact we use Formula 6 to compute the similarity of two nodes N_1 and N_2.

$$sim(N_1, N_2) = \frac{\sum_{i=1}^{3} compare(a_i, a_i')}{3} \qquad (6)$$

In Formula 6, a_i and a_i' are the ith attributes of nodes N_1 and N_2 respectively. If two nodes have the same value on a given attribute then compare function returns 1 and otherwise returns 0. Of course comparing two fields can has a value in the range of 0 to 1 in general. Here for simplicity we considered it as a function with only two returned values (zero or one).

C. *Implementation*

We implemented the proposed method in Java with four designed classes namely **PreparationGraph**, **Graph**, **Samples** and **SummaryGraph** for this matter. We designed the first class for extracting the subgraph of each entity and put vertices and edges of that entity in separate files. Based on the figures resulted from four entities, as described in next section, we decided to select **paper** subgraph for summarization. **Graph** class has methods to construct graph, getting its vertices and edges and setting attributes of vertices. An instance of Graph class constructed for **paper** graph. For experimental aim, **Samples** class has designed to make subgraphs with different sizes of **paper** graph. In fact **Samples** class get a size as input and creates a sub-graph of that size. **SummaryGraph** has designed to get an instance of Graph and provide methods to summarize it.

D. *System Configuration*

We used a system with the configuration given in Table 6 to run program and evaluate the proposed method.

Table 6: System configuration

Processor	Intel(R)Core(TM) i7 3.5 GHz
RAM	32GB
System type	64 bit
OS	Windows 8

E. *Results*

To present an overall view of number of vertices and edges of graph, we produced 12 subgraphs that the number of vertices and edges are given in Table 7 and its chart is shown in Figure 6.

To show the order of running time of the summarization algorithm, we produced some subgraphs with sizes given in Table 8 and run summarization algorithm for each of graphs with k=10. Their running times are given in Table 8 and its chart is shown in Figure 7.

Up to now the best method to summarize a graph based on both structure and concepts is SGVR [8]. Henceforth to evaluate our proposed method, we compared our method with this method. In the aim of evaluation, we implemented our proposed method and also SGVR method and summarized graphs with

Table 7: Number of edges versus number of vertices

#vertices	10	100	1000	2000	3000	4000	5000	6000	7000	8000	90000	10000
#edges	0	1	253	2335	6633	12526	19104	27618	36081	45888	57600	69159

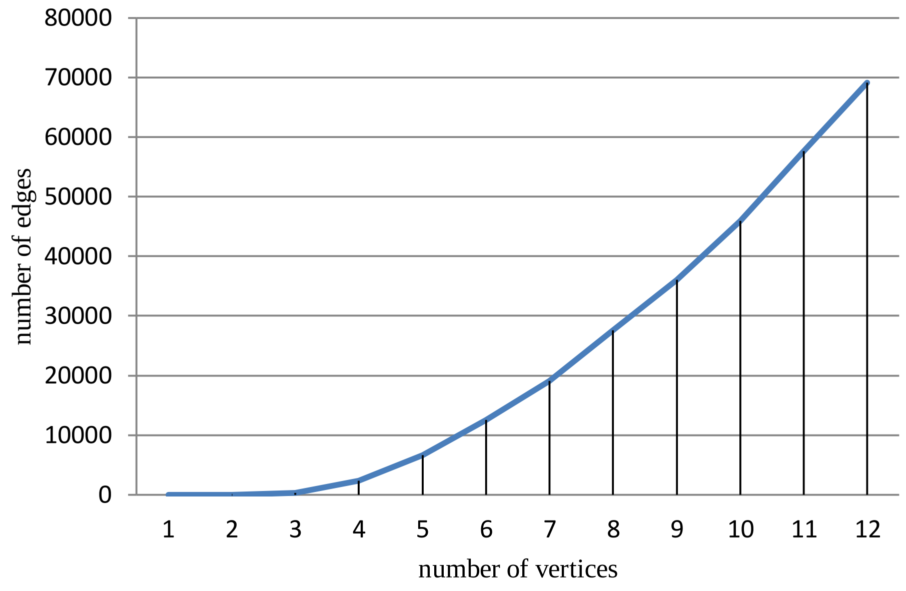


Fig 6: Number of edges based on the number of vertices

Table 8: Processing times based on graph size

Summary size (No. of vertices)	100	200	300	400	500	600
Processing time (in sec.)	2	9	60	256	796	1696

different sizes and different values of α as shown in Figure 8. The summary produced with our method and with α=0.0 is approximately equal to the summary produced with SGVR method according to Figure 8.

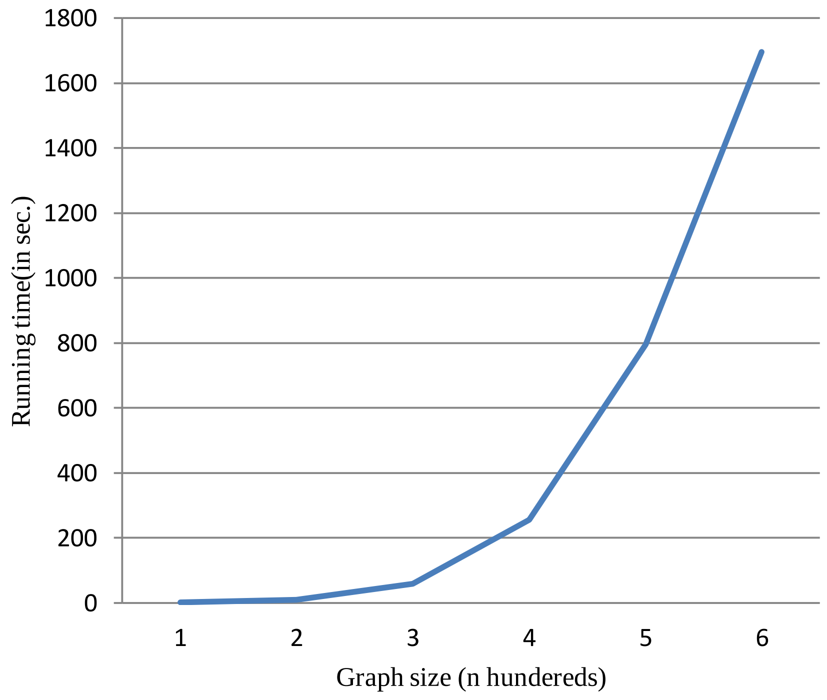
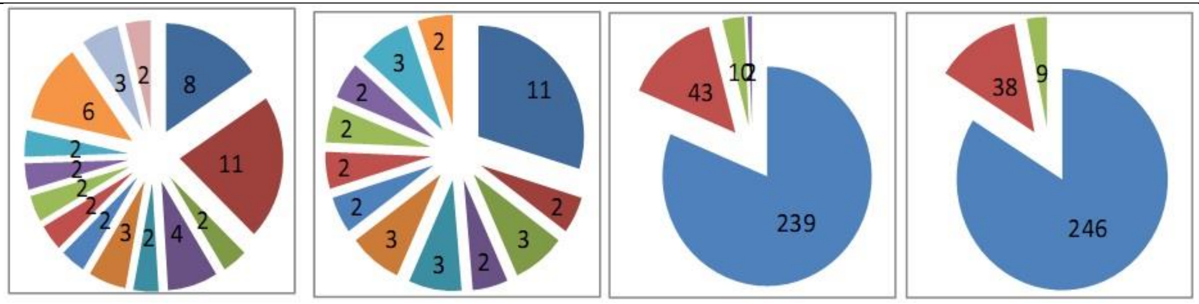


Figure 7: Running time of summarization algorithm based on graph size

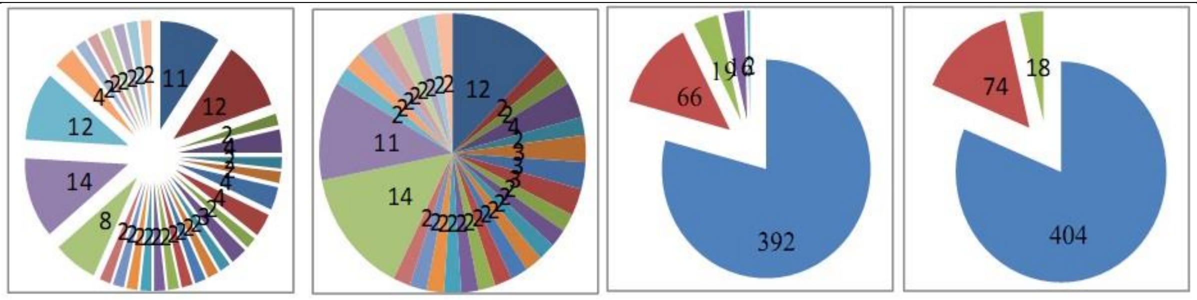
Density measurement has been used to evaluate the quality of produced summaries. The density of the summary graph is computed as follows:

$$\mathrm{den}(\{V_i\}_{i=1}^{k}) = \sum_{i=1}^{k} \frac{|\{(v_p,v_q)|v_p,v_q \in V_i\,,(v_p,v_p) \in E\}|}{|E|}$$

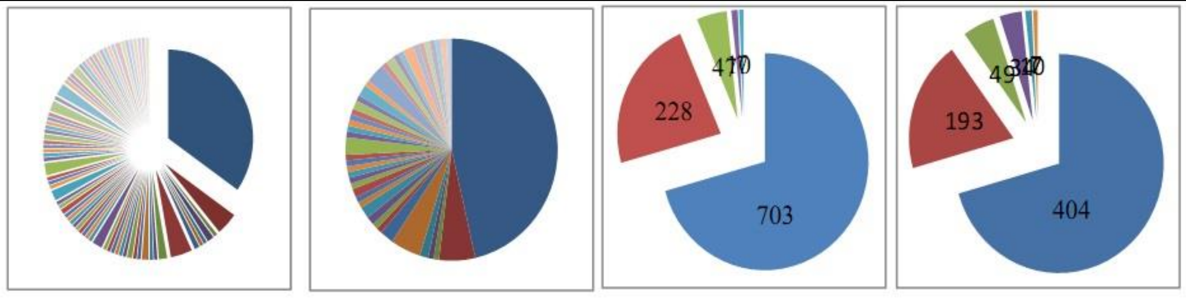
As shown in Figure 8, the density of summary produced by our proposed method for values near to 0 for α is approximately equal to density of summary produced by SGVR method.



$n_v(G)$=300 a)α =0.5 den=0.9629 b) α =0.2 den=0.5185 c) α =0.0 den=0.4814 d) SGVR den=0.5185



$n_v(G)$=500 α =0.5 den=0.9146 b) α =0.2 den=0.6829 c) α =0.0 den= 0.5365 d) SGVR den=0.5365



$n_v(G)$=1000 α =0.5 den=0.9486 b) α =0.2 den=0.7667 c) α =0.0 den=0.4189 d) SGVR den=0.4103

Fig 8: Each row shows summary of a given graph with determined size. First three columns of every row have produced by our proposed method and the fourth one by SGVR method. Each chart shows the size of super-nodes of the summary. Of-course only super-nodes with size greater than one has shown.

V. Discussion

Our experimental results shows that the contribution of structure in the summary generated by Bei's method is too small. As we see in columns 3 and 4 of Fig 8, the density of the summary generated based on Bei's method is equal to density of summary of our proposed method with the structure contribution of zero. Thus Fig 8 shows that the contribution of structure in the resulting summary based on Bei's method is close to zero. Unlike Bei's method our proposed method is flexible and can generate a summary with any contribution degree of the structure. Summaries with different sizes are resulted because of considered different values for structure contribution in our proposed method.

The proposed method has the following features that make it superior to existing methods. In our opinion this method is the best candidate to graph summarization.

- Generate a summary with any size

In methods such as Bei's generating a summary with any size is not possible, because in these methods an initial summary is created based on attributes and their values. For a real-world graph which every node have many attributes the initial summary is not small. But our proposed method works based on removing edges and creating a summary with a proportional size is possible.

- Producing summary based on user needs

The degree of contributions of structural and attribute similarities in the resulting summary can be determined by the user based on our proposed method. In our proposed method, it is possible to increase the contribution of the structure and attribute similarities in the resulting summary.

- Unified approach

Some methods like the one proposed by Bei at first summarize graph based on attribute similarities and then adjust the summary to support graph topology. Such methods may work for some graphs but they are inefficient for situations where changing the resulting attributed summary to support graph topology needs a lot of vertex exchange between super-nodes. Our proposed method considers both structure and attribute concurrently and have no additional overhead.

- Learning capability

In situations where summaries are used for query answering, the contributions of structure and concepts in producing summary can be learned by the algorithm. Therefore by producing different summaries and evaluating their accuracy in answering user queries, the weighting factors of structure and concepts to constructing summary can be learned by the algorithm.

Up to now the best method for graph summarization is SGVR method [10]. As seen in columns 3 and 4 of Figure 8, our method converge to SGVR by taking the value of α near to 0. Thus our method is more flexible than SGVR method in considering any contribution degree for structure to generating summary. Because in our method the contribution of structure can be given explicitly by the user while in Bei's method it is resulted by adjusting summary to graph structure. Thus we say that our proposed method is more general than Bei's method.

VI. Conclusion and Future Work

A new method for summarizing a graph based on both structure and concepts proposed. The proposed method implemented in Java and evaluated by real life dataset HEP-TH. We compared our method with Bei's method [10] by implementing this method also. The experimental results showed the effectiveness of our method. The proposed method has the advantage that the contributions of structure and attribute similarities can be determined by user and for this reason generate summary based on user needs.Summary graph can be used for answering user queries. The more precisely queries the summary answers, the more the summary is better. Determining the best values for contributions of structure and concepts in producing summary is important and is one of our future plan. Learning which kind of summarization, structural, conceptual or mixture of them is necessary to answer a given query set is of our future plan. Designing a set of queries to evaluate the accuracy of summarization methods is also of our future plans.

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