

Knowledge Evaluation Based on Learner Assessment and Utilizing Fuzzy System to Increase the Accuracy of Evaluation

Ahmad Kardan

Advanced E-Learning Technology Laboratory
Computer Engineering and IT Department
Amirkabir University of Technology
Tehran, Iran
aakardan@aut.ac.ir

Gamarnaz Tadayon Tabrizi

Information Technology Department
Islamic Azad University of Mashhad
Mashhad, Iran
tadayon@mshdiau.ac.ir

Seyed Mohsen Naghibizadeh Meybodi

Information Technology Department
Islamic Azad University of Mashhad
Mashhad, Iran
mohsen@naghibizadeh.com

Abdolreza Ghalamkari

Advanced E-Learning Technology Laboratory
Computer Engineering and IT Department
Amirkabir University of Technology
Tehran, Iran
areza_gh@aut.ac.ir

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Abstract—Learner knowledge evaluation is a process of decision making, personalization and adaptation of learning system. Generally, Traditional methods of knowledge evaluation use few variables in the evaluation process; For instance, the only parameter which is involved in the process is accuracy of answers. While, in case of increasing the accuracy of evaluation in this process, more variables need to be considered. Fuzzy system with ability to accept variety of input variables, inference and output generation, is a good option for the process of evaluation, such that in the recent years there are so much effort in the way of using fuzzy logic and its capabilities in knowledge evaluation that have been done so far. In this paper we will present a method to evaluate the knowledge of learner that by using fuzzy system in three phases which are fuzzification, inference engine, defuzzification with considering more variables like accuracy rate, importance and complexity of questions, performs the evaluation process. The prime features of the presented system are clarity, flexibility and simplicity in implementation.

Keywords—Knowledge Evaluation; Knowledge Assessment; E-Learning; Fuzzy Logic; Learner Modeling

I. INTRODUCTION

Today, e-Learning and virtual learning environments are playing a vital role in developed and developing countries. Many universities and business

corporations in the world benefit from the e-Learning in order to train their students and staffs [1, 2]. One of the significant points of these environments is its adaptability with the students' needs.

Therefore, the learning system always tries to gather and investigate the information of students, such as, level of the student's knowledge, interests, background and personal characteristics [3]. Due to, the attributes which are the main part of decision making in learning systems, the way of evaluation of the attributes by the system is one of the main challenges in learning system. So, considering the evaluation and finding new practical solutions in this area are one of the main fields of researchers' interests. This paper intends to review the learner knowledge evaluation based on fuzzy logic. Regard to powerful mathematics bases of Fuzzy logic in theory sets and using human understandable verbal variables, it is capable to provide the flexible environment with variety of option for evaluation [4, 5]. In the recent decade, Fuzzy theory has been used by many scientists in order to evaluate Learner's knowledge [6]-[12]. Chen and Bai offer a method for student's knowledge evaluation using fuzzy system which took variables in their account: Accuracy rate, Response time, Question importance and complexity of question and finally students were evaluated based on the final grades [10, 11]. In section 2 and 3 Adaptive Learning System and Learner modeling will be presented and section 4 is dedicated to logic and fuzzy systems. In section 5 we review the Chen and Bai method and in section 6 an appropriate method for development and implementation of the proposed system is presented. In section 7 the usage of the proposed method will be showed by giving an example. Then we compare our proposed method with Chen and Bai one and at last the conclusion is presented in section 8.

II. ADAPTIVE LEARNING SYSTEM

Regarding the issues presented in this paper an adaptive learning system is one which adapts its learning services to learner need. In another word, an adaptive learning system considers the learner model based on his/her knowledge level, learning style, backgrounds, and other needs presented by the mentioned model to decide what sort of services should be provided to learner. Therefore each individual learner or at most learners classified in groups interacts with a learning system adapted to them. It is clear that the key point for a successful adaptation depends on an accurate modeling of the learner's characteristics [15].

III. LEARNER MODELING

In the learning systems the main role is dedicated to learner, and therefore the user modeling is named as learner modeling. To model a learner in addition to his/her profile, other characteristics regarding knowledge level, background, interests, emotional behaviors, learning goals, and learning style should be captured. These features could be prepared explicitly through questionnaires and interview process or implicitly by tracking his/her activities during a learning session. One of the most popular methods for modeling the knowledge level is based on the assessment of learner's knowledge by means of appropriate tests. Some of the accessible parameters in

a test are the level of the obtained knowledge, the accuracy of responses, the time being spent, and so on [15].

IV. INTRODUCING FUZZY LOGIC AND FUZZY SYSTEM

A. Fuzzy Logic

Fuzzy logic founded by Professor Lotfali Zadeh Which is based on sets theory [14]. It is possible to imagine the sets theory as black and white figures. So that the member m is in A set or not; But we can imagine Fuzzy Logic as a gray scale. So that the member m can be part of A set as well as B set. In Fuzzy logic we deal with verbal variables and membership function which is a numeric value between 0 to 1 intervals. For example, if the height of a human considered as a variable, it is possible to consider these verbal variables: height – ("Very short", "Short", "Medium", "Tall", "Very Tall").

B. Fuzzy System

Figure 1 shows the existing components in the fuzzy system. Fuzzy system based on the design type accepts the inputs, and by using fuzzy logic executes the inference operation and generate the output. In the following we will describe the fuzzy system components.

fuzzification: convert certain input values into fuzzy values.

Fuzzy rules base: Fuzzy system consists of set of IF-THEN rules as follows:

If $X_1=A_1$ and $X_2=A_2, \dots, X_n = A_n$ Then $Y=B$

Inference Engine: By using rule base and fuzzy logic, fuzzy system can produce inference. Inference engine can operate in two ways which are: mixing the rules and separating the rules. Among the inference engines provided, we can refer to Mamdani, Multiplied and Zadeh.

Defuzzification : Create certain output values from fuzzy values.

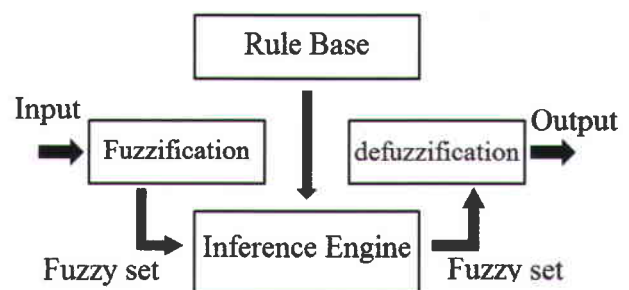


Fig .1. Fuzzy System

V. REVIEW OFFERING METHOD OF CHEN AND BAI FOR EVALUATION LEARNING SYSTEMS

In the offered Chen and Bai method, it is assumed that there are n students and m questions needed to be answered. For evaluation purposes, parameters have been used which described in table 1.

Table 1. The parameter description in the offered Chen and Bai method.

Parameter Name	Icon	Description
Accuracy Rate Answer	A	Score ratio of each students to specific question divided by best answer to the same question.
Answer Time Rate	T	Student answering period ratio to specific question divided by the longest answering time to the same question.
Question importance	P	Level of question importance based on question designer opinion as an expert person.
Question Complexity	C	Level of complexity of question based on question designer opinion as an expert person.
Question difficulty	D	This parameter shows the difficulty level of question based on accuracy rate and time to answer.
Effort	E	This parameter shows amount of effort which has been done for attempting to answer each question based on question difficulty and complexity.

Parameter accuracy rate of students answers to questions showed by matrix A as following:

$$A = [a_{ij}], m \times n \quad (1)$$

Where $a_{ij} \in [0,1]$ shows accuracy ratio of student j who answers question i. Students time rate presented by matrix T:

$$T = [t_{ij}], m \times n \quad (2)$$

Where $t_{ij} \in [0,1]$ shows the response time of student j who answers question i. a mark has been given to each of the questions; sum of them is equal to 100 and vector G can be expressed as following:

$$G = [g_i], m \times 1 \quad (3)$$

$$\sum_{i=1}^m g_i = 100 \quad (4)$$

Using matrix A and vector G total score of each student in the vector S is obtained:

$$S = A^T G = [s_j], n \times 1 \quad (5)$$

Where $s_j \in [0,100]$ represents the final mark of student j and AT is transposition of matrix A and if the matrix S is sorted in descending order, student's priorities are determined. Although, this is traditional method and there is a chance that some of the students have equal marks. For solving this issue Chen and Bai add more fuzzy variables in the form of a matrix to the system and using three steps, the fuzzy system leads itself to a path which produces a precedence sequence of student marks that don't have equal marks.

Importance of the question parameter is one of the most important factors, needed to be considered in the evaluation process. Therefore, one level of the importance degree, which is defined in fuzzy form for the questions, is presented as matrix P as follows:

$$P = [p_{ik}], m \times l \quad (6)$$

Where $p_{ik} \in [0,1]$ presents question I membership function to importance level k. In this paper five level of fuzzy importance have been used ($l=5$); Where $k=1$ till $k=5$ present the following verbal values in order: Low, More or less low, Medium, More or less high, High. Membership function values are presented in Figure 2. Similarly five fuzzy levels presented for these parameters: Answers accuracy ratio, Answers time rate, Difficulty and Complexity. The complexity of each question determines the ability of the students in giving correct answers; this is one of important factors that can be expressed by an expert and presented in Matrix C.

$$C = [c_{ik}], m \times l \quad (7)$$

Where $c_{ik} \in [0,1]$ presents membership function value of question i belong to complexity level k.

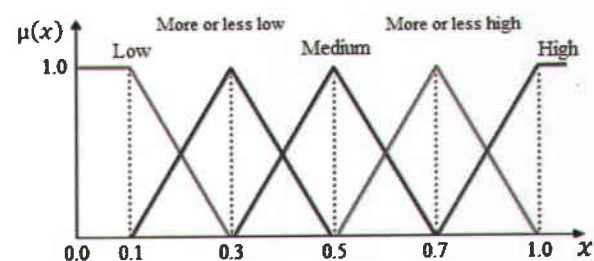


Fig. 2. Fuzzy membership function for five levels

Chen and Bai method evaluate learner knowledge in three steps. In the first step absolute values obtained from the rate of accuracy and response time rates are achieved where $\bar{A}$ represents the average accuracy rate and can be calculated as following:

$$\bar{A} = [a_i], m \times 1 \quad (8)$$

Where a_i represents average accuracy ratio of answers to question I and n is number of students where:



$$a_{i.} = \sum_{j=1}^n a_{ij}/n \quad (9)$$

And $\bar{T}$ represents average response time value and presented as below:

$$\bar{T} = [t_{i.}], m \times 1 \quad (10)$$

Where $t_{i.}$ represents average response time rate to question I which is equal to:

$$t_{i.} = \sum_{j=1}^n t_{ij}/n \quad (11)$$

In the next step by using accuracy fuzzy ratio (FA) and Fuzzy response time rate (FT) in the fuzzification matrix we have:

$$FA = [fa_{ik}], m \times l \quad (12)$$

$$FT = [ft_{ik}], m \times l \quad (13)$$

Moreover, taking to an account of Matrix FA and FT and R_D fuzzy rules that they are in the form of IF-Then, question difficulty matrix D can be obtained as following:

$$D = [d_{ik}], m \times l \quad (14)$$

Where $d_{ik} \in [0,1]$ represents the difficulty membership function value of question I regarding to fuzzy level k. with R_D law enforcement on the level of accuracy l_A and response time l_T level of tough questions l_D level can be presented as below:

$$l_D = R_D(l_A, l_T) \quad (15)$$

By applying the weights w_A and w_T and using the following formula values come from the matrix D can be obtained (Note that all weights are expressed in articles are identified by an expert):

$$d_{ik} = \text{MAX} \{w_A \cdot fa_{i,l_A} + w_T \cdot ft_{i,l_T}\} \quad (16)$$

Then, according to the matrix D (difficulty of questions) that was obtained in the previous step and the matrix C (complexity of questions) that are marked by an expert and using weights that w_C and w_D that their total value is equal to 1 and applied fuzzy rules matrix R_E (Effort matrix), E is obtained:

$$E = [e_{ik}], m \times l \quad (17)$$

Where $e_{ik} \in [0,1]$ represents effort membership function value of each student in response to question i regarding to fuzzy level k. Then using the matrix E (effort of students), the matrix P (question importance), using weights w_P and w_E that their total is equal to 1 and enforce the fuzzy law R_W compatibility matrix, W is obtained:

$$W = [w_{ik}], m \times l \quad (18)$$

Where $w_{ik} \in [0,1]$ Expresses membership function value of question i related to fuzzy level k. nineteenth formula is to get the adjustment vector is used:

$$\bar{W} = [w_i], m \times 1 \quad (19)$$

Where $w_i \in [0,1]$ represents the final adjustment amount required for question i and it is calculated as following:

$$w_i = \frac{0.1 \times w_{i1} + 0.3 \times w_{i2} + 0.5 \times w_{i3} + 0.7 \times w_{i4} + 0.9 \times w_{i5}}{0.1 + 0.3 + 0.5 + 0.7 + 0.9} \quad (20)$$

Numbers 0.1, 0.3, 0.5, 0.7 and 0.9 are centers of triangular membership functions in Figure 2. Note the expressed formula (16) developed by Chen and Bai and it is not a known inference engine.

In the final part by using the formula (21), the final grade of students who have same grade on traditional assessment, can be calculated. All variables in this formula are known. q Represents the number of students where in traditional assessment obtained equal grade. M is equal to the number of questions. Value of w_i was calculated by the Formula (20). a_{ij} Value is equal to the accuracy response ratio to question j by student i.

$$SOD_j = \sum_{k=1}^q \sum_{i=1}^m (a_{ij} - a_{ik}) \cdot (0.5 + w_i) \quad (21)$$

VI. THE PROPOSED METHOD FOR STUDENT KNOWLEDGE EVALUATION

In the previous section we observed that the proposed method of Chen and Bai totally forms 7 value as weight of W which is determined by the individual expert using the calculation (weights w_A , w_T , w_D , w_C , w_E , w_P and fixed weight 0.5 in formula 21). This is one of the disadvantages of their proposed system; because such a weight determination requires precision and skill of special expert and any change in the source can change the system result. On the other hand their proposed method does not benefit from a famous inference engine because it must balance weights and this jeopardizes the fuzzy system understanding.

In the proposed system Weights has been removed and the system uses a total of three fuzzy subsystems. Each of the following three systems, a fuzzy system with two independent inputs, and an inference engine are fuzzification and defuzzification rules base maker. In the proposed method, we used Mamdani inference engine [13] and Center Of Gravity (COG) defuzzification have been used. Figure 3 shows outline of the proposal.

Each of the three phases includes the following three steps:

First phase (Fuzzification): At this stage, certain values, obtained by getting the value of membership function, are converted to fuzzy values. Similar to Figure 1, this stage simplifies the calculations the fuzzy triangular maker has been used.



Second phase (Inference): in this step, the system uses Mamdani inference engine maximum - minimum [13] and IF - THEN law to perform the inference. The inference engine can be presented in the form of following formula:

$$\alpha_{ik} = \text{MAX}_{\{(l_1, l_2) | R(l_1, l_2) = k\}} \{\min(fa_{i, l_1}, ft_{i, l_2})\} \quad (22)$$

Where α_{ik} inferences output of question I in fuzzy set k and fa_{i, l_1}, ft_{i, l_2} are fuzzyfied value of accuracy ratio, time rate and response time which result in generating the following matrix:

$$\alpha = [\alpha_{ik}], m \times l \quad (23)$$

Third phase (defuzzification): In this section, the fuzzy output values turn into final values. In this paper we used exert center method for defuzzification which can be expressed in the following formula:

$$y_i = \int x \cdot \mu(x) dx / \int \mu(x) dx \quad (24)$$

Each of the three systems expressed in Figure3 are doing the presented phases. Fuzzy system in the first step takes the accuracy ratio and the time as inputs and generate question difficulty matrix D; where by combining it with question complexity matrix C, the second step input forms; output of second step will be students' effort matrix E along with the importance matrix P forms the input of third step input and finally the third step output will be adjustment vector W.

At last, Following formula can generate Vector $\tilde{G}$ from known vector G and the adjustment value obtained from the system.

$$\tilde{G} = [\tilde{g}_i], m \times 1 \quad (25)$$

$$\tilde{g}_i = g_i \cdot (1 + w_i)$$

Then, using the formula (26), we scale up value vector $\tilde{G}$ in a way that the total amount will be equal to 100.

$$\tilde{g}_i = \tilde{g}_i \cdot \sum_{j=1}^m g_j / \sum_{j=1}^m \tilde{g}_j \quad (26)$$

Finally, the students' final grades are obtained by the following formula in descending order; the order of priority of students' assessment based on their knowledge evaluation grades will be obtained.

$$\tilde{S} = A^T \tilde{G} \quad (27)$$

VII. A SAMPLE USAGE OF PROPOSED METHOD

In this section, by providing examples, the usage of proposed method is shown and in the end of this section, the proposed method is compared with the results of Chen and Bai methods.

We assume a professor in physics subject plans to assess his students. He selected 10 students $s_1, \dots, s_{10}$ to answer 5 questions $Q_1, \dots, Q_5$. Questions have been proposed as multiple choice questions so that each question can have one or more than one correct answer. Matrices P and C have been initialized by a designer as individual expert, which respectively, represent the level of importance and the level complexity of the questions in his perspective. Also vector G represents the grade of each of the questions that the total sums of all questions grades are equal to 100. After the students responded to questions, A and T matrices obtained that matrix A represents the accuracy rate of student answers (the grade of each student answer to the specific question grade ratio divided by the best answer to the question) and the matrix T expresses the rate of student response time to questions (student answers time to a specific question ratio divided by the longest time to answer the same question).

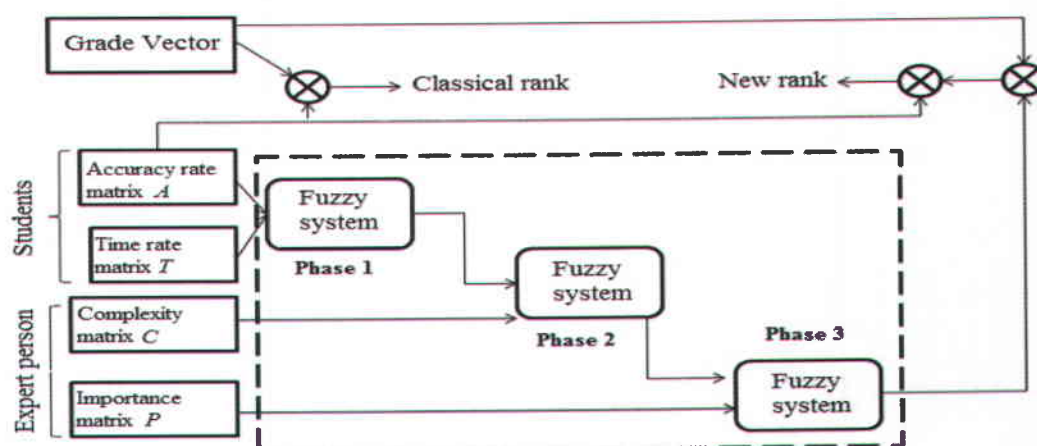


Fig.3. Outline of the proposed fuzzy system

$$A = \begin{matrix} & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{bmatrix} 0.59 & 0.35 & 1.00 & 0.66 & 0.11 & 0.08 & 0.84 & 0.23 & 0.40 & 0.24 \\ 0.01 & 0.27 & 0.14 & 0.04 & 0.88 & 0.16 & 0.04 & 0.22 & 0.81 & 0.53 \\ 0.77 & 0.69 & 0.97 & 0.71 & 0.17 & 0.86 & 0.87 & 0.42 & 0.91 & 0.74 \\ 0.73 & 0.72 & 0.18 & 0.16 & 0.50 & 0.02 & 0.32 & 0.92 & 0.90 & 0.25 \\ 0.93 & 0.49 & 0.08 & 0.81 & 0.65 & 0.93 & 0.39 & 0.51 & 0.97 & 0.61 \end{bmatrix} \end{matrix}$$

$$T = \begin{matrix} & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{bmatrix} 0.7 & 0.4 & 0.1 & 1 & 0.7 & 0.2 & 0.7 & 0.6 & 0.4 & 0.9 \\ 1 & 0 & 0.9 & 0.3 & 1 & 0.3 & 0.2 & 0.8 & 0 & 0.3 \\ 0 & 0.1 & 0 & 0.1 & 0.9 & 1 & 0.2 & 0.3 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0 & 1 & 1 & 0.3 & 0.4 & 0.8 & 0.7 & 0.5 \\ 0 & 0.1 & 1 & 1 & 0.6 & 1 & 0.8 & 0.2 & 0.8 & 0.2 \end{bmatrix} \end{matrix}$$

$$G = \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} \begin{bmatrix} 10 \\ 15 \\ 20 \\ 25 \\ 30 \end{bmatrix}$$

$$P = \begin{matrix} & P_1 & P_2 & P_3 & P_4 & P_5 \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.85 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.07 & 0.93 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$C = \begin{matrix} & CS_1 & CS_2 & CS_3 & CS_4 & CS_5 \\ \begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{matrix} & \begin{bmatrix} 0 & 0.85 & 0.15 & 0 & 0 \\ 0 & 0 & 0.33 & 0.67 & 0 \\ 0 & 0 & 0 & 0.69 & 0.31 \\ 0.56 & 0.44 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 \end{bmatrix} \end{matrix}$$

If the traditional method like formula 5 for evaluation of student be used, students $s_1, \dots, s_{10}$, respectively, obtained the following results:

$$\begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} \\ 67.6 & 54.05 & 38.4 & 49.7 & 49.7 & 48.8 & 46.1 & 52.3 & 85.9 & 49.7 \end{matrix}$$

We can observe that the grade of three student s_4, s_5, s_{10} are equal. Because only the accuracy answers ratio matrix is used to assess responses. Now, using the proposed method in this paper we assess students. As described in the previous section, the proposed method contains three phases each of which include three steps: fuzzification, inference, and defuzzification. What is more, each of phases and steps will be dealt for solving the above example.

Phase 1:

First step (Fuzzification): in this step, fuzzification operation executes on Matrices T and A. As described, for the simplification purpose, the calculations of triangular fuzzification (figure 2) have been used and like Chen and Bai methods formula (12) and (13) have been used which result in a fuzzy matrix FA and FT is produced.

$$FA = \begin{bmatrix} 0 & 0.25 & 0.75 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0.945 & 0.055 \\ 0 & 0.15 & 0.85 & 0 & 0 \\ 0 & 0 & 0.315 & 0.685 & 0 \end{bmatrix}$$

$$FT = \begin{bmatrix} 0 & 0 & 0.65 & 0.35 & 0 \\ 0 & 0.1 & 0.9 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.65 & 0.35 & 0 \end{bmatrix}$$

Second step (inference): Using Mamdani inference engine in formulas (22) and inference rules and input matrices FA and FT, fuzzy difficulty questions matrix α_D is generated. α_{14D} expresses the value of the first row and fourth columns of the matrix; α_{14D} is calculated as follows:

$$\alpha_{14D} = \max \{ \min(0, 0), \min(0, 0.65), \min(0.25, 0.65), \min(0.25, 0.65), \min(0.75, 0.35), \min(0.75, 0), \min(0, 0) \} = 0.35$$

Using the same pattern the rest of the matrix elements α_D is calculated:

$$\alpha_D = \begin{bmatrix} 0 & 0 & 0.65 & 0.35 & 0 \\ 0 & 0.05 & 0.1 & 0.9 & 0 \\ 0.55 & 0.945 & 0 & 0 & 0 \\ 0 & 0 & 0.85 & 0.15 & 0 \\ 0 & 0.65 & 0.35 & 0.315 & 0 \end{bmatrix}$$

Third step (Defuzzification): At this step, making use of the exert center according to Formula (24), fuzzy matrix α_D becomes non-fuzzy vector D:

$$D^T = [0.576 \quad 0.653 \quad 0.299 \quad 0.538 \quad 0.456]$$

Phase 2:

First step (Fuzzification): At this step, the matrix D using the triangular fuzzification becomes fuzzy matrix FD:

$$FD = \begin{bmatrix} 0 & 0 & 0.622 & 0.378 & 0 \\ 0 & 0 & 0.236 & 0.764 & 0 \\ 0.003 & 0.997 & 0 & 0 & 0 \\ 0 & 0 & 0.811 & 0.189 & 0 \\ 0 & 0 & 0.779 & 0 & 0 \end{bmatrix}$$

Second step (inference): Using Mamdani inference engine formulas (22), inference rules and input matrices FD and C, fuzzy efforts matrix α_E is obtained:

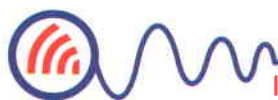
$$\alpha_E = \begin{bmatrix} 0 & 0.622 & 0.378 & 0.15 & 0 \\ 0 & 0 & 0.236 & 0.67 & 0 \\ 0 & 0.003 & 0.994 & 0.31 & 0 \\ 0 & 0.56 & 0.189 & 0 & 0 \\ 0 & 0.221 & 0.7 & 0.3 & 0 \end{bmatrix}$$

Third step (Defuzzification): In this section, using exert center defuzzification matrix, α_E becomes non-fuzzy vector E:

$$E^T = [0.424 \quad 0.642 \quad 0.568 \quad 0.354 \quad 0.514]$$

Phase 3:

First step (Fuzzification): At this stage by using triangular fuzzification, vector E converted to fuzzy matrix FE:



$$FE = \begin{bmatrix} 0 & 0.38 & 0.62 & 0 & 0 \\ 0 & 0 & 0.289 & 0.711 & 0 \\ 0 & 0 & 0.661 & 0.339 & 0 \\ 0 & 0.733 & 0.267 & 0 & 0 \\ 0 & 0 & 0.931 & 0.069 & 0 \end{bmatrix}$$

Second step (inference): Using Mamdani inference engine formulas (22), inference rules and input matrices FE and P, fuzzy efforts matrix α_W is obtained:

$$\alpha_W = \begin{bmatrix} 0 & 0 & 0 & 0.62 & 0 \\ 0 & 0.289 & 0.33 & 0.67 & 0 \\ 0 & 0 & 0 & 0.661 & 0.339 \\ 0.733 & 0.267 & 0 & 0 & 0 \\ 0 & 0.07 & 0.93 & 0.069 & 0 \end{bmatrix}$$

Third step (Defuzzification): In this section, using COG defuzzification matrix α_W becomes non-fuzzy vector W:

$$W^T = [0.7 \quad 0.552 \quad 0.749 \quad 0.177 \quad 0.5]$$

Now using the matrix W and Formula (25) and (26) assessment matrixes $\tilde{G}$ is obtained:

$$\tilde{G}^T = [11.371 \quad 15.566 \quad 23.296 \quad 19.675 \quad 30.092]$$

Finally, new assessment of students' grades using formula (27) is obtained:

$$\hat{S}^T =$$

S ₁	S ₂	S ₃	S ₄	S ₅	S ₆	S ₇	S ₈
67.15	53.17	42.1	52.19	48.31	51.81	48.47	49.27
S ₉	S ₁₀						
85.23	51.49						

It can be observed that in order of preference among the three student s_4, s_5, s_{10} which their grades were equal, now changed as following:

$$S_4 > S_{10} > S_5$$

Thus, the output of the proposed algorithm is equal to the result obtained in Chen and Bai methods. With the difference that our proposed method uses Mamdani inference engine and mentioned weight list in the method of Chen and Bai does not exist and this simplifies fuzzy understanding of the system. When Chen and Bai method is applied on all students' data, respectively the following results obtained in order for students s_1 to s_{10} :

S ₁	S ₂	S ₃	S ₄	S ₅	S ₆	S ₇	S ₈
139.8	-7.9	-145.1	-41.8	-70.9	-51.8	-72.9	-41.7
S ₉	S ₁₀						
337.8	-45.4						

The result creates the students' grades in order and from two aspects which have differences with the proposed method on this paper:

1. From final point of view of obtained grades, the existed weights in Chen and Bai method emphasis on some of the parameters and change the values more or less.
2. Due to, Final grades obtained in the proposed method, scalability operation in Formula* (26) is

balanced and it is in the range 0 to 100, while the final obtained grades in the method of Chen and Bai doesn't have specific range (in this example, the obtained grades Range is -145.1 to 337.8). This issue is another disadvantage of Chen and Bai system.

Future Works

As future works, we would like to investigate and examined both presented systems in this paper on the student population. For this reason, course selection and also design of the questions should be based on fuzzy system which is crucial. In courses, the way of response methods are not descriptive and it is possible to create multiple choice questions; because, it is simpler to make them as fuzzy membership and to put them in the range of 0 to 1 and as a result accuracy rate response to each question will be achieved.

Another way to obtain accurate ratio is to calculate the best answer of desired question and divide the other answers marks to the best answer mark.

To calculate the response time rate a scheduler can also be used such that the longest response time of the question needed to be considered and divide the rest of times to it.

VIII. CONCLUSION

In this paper a method is proposed to evaluate learner knowledge using fuzzy systems. Proposed method using fuzzification, inference engine, and defuzzification with respect to variables such as difficulties, importance and complexity of the question, perform the evaluation process. Clarity, flexibility and simplicity in implementation, are the bold features of our proposed method in this article. In section five we tried to explain the usage of proposed method by providing an example of student evaluation and we compared it with Chen and Bai method. It is observed that the proposed method eliminates the used weights in the previous method because the dependencies of inference operation to weights can cause inexcusable changes in the results. Unlike previous methods, the proposed method performs inference operations because it uses famous inference engines such as Mamdani inference engine which provides more fuzzy understanding. Finally, the proposed method balances the student's evaluation meaning against previous methods; for instance if the total score of question is 100 the final score of student result is also located in the range 0 to 100.

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Ahmad A. Kardan received his B.Sc. in Electrical Engineering from Sharif University of Technology (1976-Iran), his M.Sc. in Digital Systems from the Brunel University (1997-UK), and his PhD. in Bio-Electric Engineering from Imperial College of Science and Technology (2001-UK). He is currently

a faculty member of the Computer Engineering Department, at Amirkabir University of Technology, Tehran, Iran. He Founded the Virtual Education Center of Amirkabir University of Technology in 2002. Dr. Kardan is involved in researches regarding Intelligent Tutoring Systems (ITS), Learning Advisory Systems, Adaptive and Self-regulated Learning, Learner Modeling, Concept Mapping, and Recommender Systems for E-learning Environments. He has presented more than 80 papers at national and international conferences, journals and also as chapters for related books.



Ghamarnaz Tadayon Tabrizi received her B.Sc. in Computer-Software Engineering from Islamic Azad University of Mashad-Iran in 1995, her M.Sc. in Software Engineering from Islamic Azad University, South Tehran Branch in 2000. She is Ph.D. student in Software Engineering, at Islamic Azad

University, Science and Research Branch. She is currently a faculty member of the Information Technology

Department, at Islamic Azad University of Mashad, Iran. She is involved in researches regarding Speech Recognition, Cellular Automata and Learning Automata. She has more than 25 papers at national and international conferences and journals.



Seyed Mohsen Naghibizadeh Meybodi received his Diploma in Mathematics and Physics from "Nemoone Dolati Maleksabet" high school in Yazd. He received his B.Sc. in Computer Engineering from Meybod Azad Islamic University in 2005. Currently he is doing his M.Sc. in Mashhad Azad Islamic University.

He is involved in researches regarding Adaptive Learning Systems and Evaluation of Learner's Achievement Based on Soft Computing. He professionally provides information management supports for the firms in the city of Yazd in central Iran.



Abdolreza Ghalamkari received his Diploma in Mathematics and Physics from Shahid Mohammad Montazeri High School in 1992 and his B.Sc. in Computer Engineering (Hardware) from Islamic Azad University (2001-Iran). He is currently studding IT Engineering

and Management at Amirkabir University of Technology, Tehran, Iran. Abdolreza Ghalamkari is involved in researches regarding Students Clustering in E-Learning Environments, Data Mining and Fraud Detections.