A New Method for SLM-Based OFDM Systems 
without Side Information

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Abstract—One of the most well-known techniques to reduce the problem of PAPR in orthogonal frequency division multiplexing (OFDM) systems is selected mapping (SLM). The chief drawback of this method is transmission of several additional bits, side information (SI), for each data block. Such side information causes bandwidth efficiency to be decreased; in addition, incorrect detection of SI in the receiver side make whole data block be lost. In this paper, we propose a technique by which side information bits are not explicitly sent. We exhibit the example of our method for an OFDM system by using 16-QAM modulation. It is shown that our proposed scheme, from the view point of bit error rate, probability of detection failure and PAPR reduction, performs very well.

Keywords-Orthogonal frequency division multiplexing (OFDM); peak-to-average power ratio (PAPR); selected mapping (SLM); side information

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is one of the most popular access methods for many wireless digital communications. It can adapt high data rate links [1-4] over harsh multipath fading. On the other hand, the time-domain OFDM signal presents a large peak-to-average power ratio (PAPR) at the transmitter’s output which is one of the challenging issues in this system. High PAPR takes out the power amplifier from linear region and reduces the average power of the amplifier output at the transmitter side [5-7]. In addition, It causes nonlinear distortion and imposes alteration in the signal constellation. To decrease the PAPR, plenty of methods have been suggested [8-14], one of the most well-known of which is Selected mapping (SLM) method. Because it is able to achieve better PAPR without distortion. In SLM, the transmitter generates multiple signals which all represent alike information through multiplying the input data by predetermined phase sequences [15]. Afterwards, the signal with the

Fig. 1. Random number cycle for QCG(4,5,3,8).
lowest PAPR is chosen for sending. In this method, it is required to be sent the selected phase sequence index, named side information (SI), by transmitter to express which candidate is selected and recovering of data block becomes possible by receiver. The transmitted SI bits cause bandwidth efficiency to be decreased; besides, when SI is incorrectly detected by receiver, all received frame will be lost. Hence, using of channel coding is needful which results in system complication and sacrifices data rate. However, for eliminating the explicit transmission of side information some methods have been introduced [16-21]. In [21], channel estimation and PAPR reduction are combined in which channel estimation is performed by selecting the place of pilot tone employed inside each data block, and then power distinction between data symbols and the pilot tone is used in order to recover the SI in the receiver side. In this paper, we leave out the pilot tones and replace them with symbols existing inside the data block. We propose a new SLM scheme in which Quadratic Congruential Generators (QCG) is used and enables receiver to recover data without explicit SI bits. The basic idea in our proposed technique is to fit the side information into transmitted symbols based on which some special locations in the transmitted data block are expanded, i.e. some transmitted symbols are extended. At the receiver side, it is tried to discover the places of the expanded symbols to recover SI index. Because, in our scheme, discovering the locations of expanded symbols corresponds to finding the SI index.

The remainder of this paper is organized as follows: In Section II, the Quadratic Congruential Generators (QCG) is introduced. In Section III, we describe the transmitter of proposed SLM. Design of receiver in the proposed scheme is introduced in section IV. Following that, Section V is devoted to performance evaluation of our proposed method. Eventually, the paper is concluded in Section VI.

II. REVIEW OF QUADRATIC CONGRUENTIAL GENERATOR (QCG)

Quadratic Congruential Generator (QCG) is a method of generating pseudo-random numbers. In QCG, the next value is computed repetitively from the prior value based on a recurrence relation of the form (1) [22]:

\[ Y_i = (a \times Y_{i-1} + b \times Y_{i-2} + c) \mod m \]  

where \( a, b, \) and \( c \) are named QCG parameters shown as QCG\((a,b,c,m)\). Let \( Z_m = \{0,1,\ldots,m-1\} \). The modulus \( m \) is a positive number ( \( m > 0 \) ) and \( a,b,c \in Z_m \). In this generator, \( Y_0, Y_1 \in Z_m \) is the seed and \( Y_i \) is the sequence of pseudo random numbers. However, in order to produce \( Y_i \) with full period it is needed to employ well-chosen QCG parameters in recurrence relation (1). In QCG, the maximum period is equal to \( m \) and the full period for all seed values is gained if and only if:

1) \( m \) is a power of 2.
2) The value of \( a \) is an even number and the value of \( c \) is an odd number.
3) \((b-a) \mod 4 = 1\).

We explain this matter by an example. By considering \( a = 4, b = 5, c = 3 \) and \( m = 8 \) it is clear that chosen parameters satisfy full period conditions. According to relation (1) and by using the supposed seed \( Y_0 = 2 \) the sequence of integers produced by this algorithm is:

\[ 2, 5, 0, 3, 6, 1, 4, 7, 2, 5, 0, 3, 6, 1, 4, 7, 2, 5, 0, \ldots \]

In Fig. 1, the random number cycle for QCG\((4,5,3,8)\) has been shown. As it is seen, the period of produced sequence is exactly equal to modulus \( m=8 \). By choosing any seed from the interval \([0 \ 7]\) \( Y_0 \in Z_m \), abovementioned sequence will be just cyclically shifted. Hence, we can consider any number on the Fig. 1 as the seed and by proceeding clockwise starting from supposed seed \( Y_0 \) the QCG sequence is produced.

Based on the above subjects, by considering QCG parameters satisfying full period we can indicate that:

The number of possible seeds \( Y_0 = \text{Period of QCG sequence} = m \)  

Note 1: Hereafter, when QCG is used the purpose is QCG with full period.

Note 2: All over this paper, we use marking in the shape of \( G^{(r)} \) to express a vector \( G \) is composed of \( R \) scalar quantities.

III. DESIGN OF TRANSMITTER IN THE PROPOSED SCHEME

Suppose an OFDM system with \( N \) subcarriers which \( N \) complex data symbols \( x_n \) are transmitted concurrently on subcarriers; in addition, a data block is represented in the form of \( X^{(N)} = \{x_0, x_1, \ldots, x_{N-1}\} \). In SLM, with the aim of generating a sum of various OFDM signals, the original data is multiplied by \( L \) predetermined phase vectors.

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**Fig. 2.** Block diagram of SLM technique for PAPR reduction.

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**Note:** The image content does not directly link to the text, and hence, is not transcribed here.
\( F_{i}^{(N)} = f_{i,n} = [f_{i,0}, f_{i,1}, \ldots, f_{i,N-1}] \), \( l \in \{0,1,\ldots,L-1\} \) which \( F_{l} \) is composed of \( N \) complex numbers \( f_{l,n} \) and \( |f_{l,n}| = 1 \) where \(|.|\) denotes the absolute value operator. So, \( L \) various vectors \( X_{i}^{(N)} = x_{i,n} \) are produced where \( x_{i,n} = f_{l,n} \times x_{i} \). After passing through inverse discrete Fourier transform, the vector \( X_{i}^{(N)} \) consisting of \( N \) symbols \( x_{i,n}, q \in \{0,1,\ldots,N-1\} \) is produced where:

\[
x_{i,n} = \frac{1}{N} \sum_{n=0}^{N-1} x_{i,n} e^{j2\pi nq/N}
\]

(3)

Among the \( L \) vectors \( X_{i}^{(N)} \), the one with the lowest PAPR is selected for sending. That is, the SLM vector generating the OFDM waveform with minimum PAPR is selected for sending. Fig. 2 depicts a block diagram of the SLM method. In order to discover the original data block \( X \) at receiver, the number of \( |\log_{2}N| \) SI bits must be sent by transmitter. The SI bits specify the special phase vector having minimized the PAPR among the \( L \) phase sequences.

In this paper, we propose a method based on the QCG algorithm. In our method, after generating the numbers producing by QCG algorithm each integer value is converted into the binary number having the number of \( \gamma = |\log_{2}N+1| \) bits, and then, all of the binary numbers are successively arranged. Hereafter, this corresponding binary sequences is named \( \text{BIN}_QCG(a,b,c,m) \). For example, in Fig. 3 the sequences of integers produced by QCG(4,5,3,8) for all possible seeds \( Y_{0} \) have been shown. Selected parameters in Fig. 3 provide all conditions to gain the full period for any amount of \( Y_{0} \). Also, \( \text{BIN}_QCG(4,5,3,8) \) has been shown in Fig. 4. As it is obvious, each corresponding binary amount contains the number of \( |\log_{2}N+1| = 3 \) bits. So, the \( \text{BIN}_QCG \) sequence includes 3*8=24 bits. One of the chief property of \( \text{BIN}_QCG(a,b,c,m) \) sequence is that in this sequence:

The number of 1’s = The number of 0’s = \( (m^{\gamma} - 1) / 2 = m / 2 \times |\log_{2}m+1| \)

(4)

In our scheme, the phase vectors \( F_{i}^{(N)} = f_{i,n} = [f_{i,0}, f_{i,1}, \ldots, f_{i,N-1}] \) are such that, for each \( F_{i}^{(N)} \), the absolute value of some elements are set to a constant value \( K > 1 \), while the absolute value of the other elements are set to the unit. Note that the places whose absolute values are \( K > 1 \) are not identical in the phase vectors \( F_{i}^{(N)} \). That is, no two separate vectors \( F_{i}^{(N)} \) can be united with identical \( K \) places. These locations, in general, are locations of 1’s in the \( \text{BIN}_QCG(a,b,c,m) \) sequences and are represented by \( \lambda \). For example, if a supposed phase vector \( F_{i}^{(N)} \) is associated with \( \lambda = \{3,6,7,9\} \), it indicates that for phase vector \( F_{i}^{(N)} \) only the elements existing in locations \( n=3,6,7 \) and 9 have the absolute value equal to \( K > 1 \); in other words, \( \lambda \) shows the SI index. So, for a given phase vector \( F_{i}^{(N)} \) there is one and only one associated \( \lambda \) and vice-versa. The phases of elements \( f_{l,n} \), as in conventional SLM, are set to any random values. In the proposed scheme, for creating the set of phase vectors we act as follows:

**Step 1:** The absolute value of elements in phase vectors \( F_{i}^{(N)} \) are set based on \( \text{BIN}_QCG(a,b,c,m) \) sequences. In other words, for supposed QCG parameters \( (a,b,c,m) \) and different seeds \( (Y_{0}) \) all of the possible \( \text{BIN}_QCG(a,b,c,m) \) sequences are produced each of which has the length of \( m^{\gamma} \). From abovementioned, it is concluded that in the proposed scheme:

The number of phase vectors \( F_{i} = \) Length of phase vectors \( F_{i} = m \)

(5)

(6)

Fig. 4 is an example of step 1.

**Step 2:** In each vector obtained from step 1, the ones and zeros are mapped to \( K > 1 \) and 1, respectively. For a supposed phase vector \( F_{j} \), the absolute value of the elements which are set to the constant value \( K > 1 \) form a set \( \lambda \). In fact, the set \( \lambda \) shows the side information that is not clearly sent because it is fit into the vector \( F_{j} \). For example, in Fig. 5 the ones and zeros obtained from step 1 are mapped to \( K > 1 \) and 1, respectively and the phase vectors of length \( 8 \times 3 = 24 \) for the OFDM system with \( N = 24 \) subcarriers are generated. In Fig. 5, we show just the absolute value of the elements because the phases of the elements, like conventional SLM, are chosen randomly.

**Fig. 3.** Sequences of integers produced by QCG(4,5,3,8). Parameters having been chosen provide all conditions to gain the full period for any amount of \( Y_{0} \). The period of produced sequences is exactly equal to modulus \( m = 8 \). As it is seen, by changing amount of \( Y_{0} \) the generated sequences are just cyclically shifted.
In the proposed technique, for detecting the phase vectors at the receiver side, it is necessary that transmitter and receiver arrive at an agreement about the QCG parameters consisting of $a$, $b$, $c$ and $m$ values. We explain the detection algorithm in detail in Section IV. After making all of the phase vectors $F_i^{(N)}$, our proposed scheme performs like conventional SLM. That is, through an element-wise multiplication of the data block $X^{(N)}$ by each phase vector, the number of $L$ vectors $X^{(N)}_i = x_{i,n}$ with $x_{i,n} = f_{i,n}x_n$ and also $L$ corresponding $X^{(N)}_i$ are produced. Eventually, the vector having the lowest PAPR is selected for sending. All over this paper, this special vector is associated with the vectors $X^{(N)}_i$ and $F^{(N)}_i$ where

$$X^{(N)}_i = x_{i,n}, \quad F^{(N)}_i = f_{i,n}, \quad n = \{1,2,...,N\} \quad (7)$$

In the proposed method, by multiplying the data block $X^{(N)}$ with the phase vectors $F^{(N)}_i$, the average energy for each transmitted symbol raises due to the fact that $|x_{i,n}| = |f_{i,n}| |x_n|$, with $x_{i,n}$ equal to $K>1$ or 1, argues that $E[|x_{i,n}|^2] > E[|x_n|^2]$, where $E[.]$ is the expectation operator.

IV. DESIGN OF RECEIVER IN THE PROPOSED SCHEME

In this paper, we investigate our scheme on a flat Rayleigh fading channel. The frequency-domain of each symbol $x_{i,n}$ after passing the flat Rayleigh fading channel is correspondent to:

$$q_{i,n} = h_n x_{i,n} + n_n \quad n = \{1,2,...,N\} \quad (8)$$

where $h_n$ is a real sample and indicates the fading affecting the $n$th subcarrier and $n_n$ is a complex sample and exhibits complex Gaussian noise with zero mean and variance $\sigma^2$. In our proposed technique, the receiver side knows that for each $N$-sample vector $(N = m \times \log_2^{{m+1}} + 1 = m \times \gamma)$ inside the received data block, some symbols have been expanded by a factor $K$ and other symbols have not been expanded. We assume that at the receiver side the fading samples are perfectly known; that is, perfect channel state information (CSI) is considered. By finding the locations of the expanded symbols, the receiver can recover the SI index. So, at the receiver side, in order to recover the side information it is tried to correctly discover the places of the expanded symbols. Before exhibiting the detection algorithm in the receiver side, it is necessary to explain some properties of BIN_QCG($a,b,c,m$).

Property 1 - The Window Property

By dividing a BIN_QCG($a,b,c,m$) sequence into $m$ equal parts (windows), every part (window) has the length of $\gamma = \log_2^{K+1}$ bits each of which is seen exactly once. For more explanation, consider QCG(4,5,3,8) in Fig. 4 for $Y_0 = 1$. By dividing this sequence into $m=8$ equal parts (windows), every part (window) has the length of $\gamma = \log_2^{K+1} = 3$ bits. For example, the contents of window #1 is 001, the content of the window #2 is 100, the content of the window #3 is 111, ..., and the content of the window #8 is 110. As it is seen from this example, content of each window is seen exactly once.

Property 2 - Indicator position Property

In a BIN_QCG sequence, a window of successive 1’s (all-one window) happens exactly once. Hereafter, we denote the all-one window by “indicator window”. In a BIN_QCG($a,b,c,m$) sequence, if position of “indicator window” is distinguished, positions of other binaries will become specified. For example, in Fig. 4, if “indicator window” be located in window #2, it is clear that it belongs to $Y_0 = 4$. So positions of other binaries for this window will become specified. That is,

$$100\ 111 \ 010101000111110001$$

Lemma 1 - Z-locations

From property 2 it is concluded that in a BIN_QCG($a,b,c,m$) sequence, if position of “indicator window
window” is distinguished, the locations of zeroes (Z-locations) will become specified. For example, in Fig.4 if “indicator window” be located in window #2, it is clear that it belongs to Yg = 4. Positions of zeros for this window have been specified by ”x”:

```
-xx 111 x-x-x-x-x-x-x-x-x-x-x-x-
```

In the proposed scheme, SI detection by receiver is made of the following stages (see fig. 6):

I. According to property 1, one of the windows of length γ (among the m extracted windows in $\text{BIN_QCG (a,b,c,m)}$) is selected as the “indicator window”.

II. According to the Lemma 1, the locations of zeroes (Z-locations), based on “indicator window” location, will become specified.

III. The energy of samples in the received frame and fading frame are calculated as (9)

$$Q_n = |h_n|^2 \quad n \in \{1,2,...,m \cdot \gamma = N\}$$

$$H_n = (h_n)^2$$

where $Q_n$ is energy of received frame samples and $H_n$ is energy of fading samples.

IV. Divide the received frame of length $N = m \cdot \gamma$ into m equal windows (w). Therefore, the number of m windows of length γ are extracted. That is, $w_i^{(\gamma)} i \in \{1,2,...,m\}$. It is attempted to find the window to which the ”indicator window” in stage I is attributed.

V. By supposing each of the windows $(w_i^{(\gamma)})$ as “indicator window” mentioned in stage I, we calculate the average energy of the symbols that exist in the Z-locations:

$$\eta_j = \frac{E[Q_j] - \sigma^2}{E[H_j]} \quad j \in \{1,2,...,m\}$$

Where $E$ is Expected Value, $Q_z$ is energy of the Z-locations in the received frame based on $w_j^{(\gamma)}$ position and $H_z$ is the energy of the Z-locations in the fading frame based on $w_j^{(\gamma)}$ position. Hence, the computation of (10) for each windows $(w_j^{(\gamma)})$ allows us to determine the one with the lowest average energy of Z-locations; that is, lowest $\eta_j$. As a result, the value of $j$ as well as $w_j^{(\gamma)}$ position is determined and we can find the real location of the ”indicator window”. Also, According to the Property 2, locations of other binary numbers will become specified.

VI. By mapping the ones and zeros to K >1 and 1 respectively, we can guess the phase vector. Such evaluations may cause erroneous detection of the index ”V”, therefore, in this case the receiver cannot discover the SI index correctly. However, the system designer must be aware that for a constant m and a larger amount of K, a larger increase of the energy can be achieved. This increase of energy causes the error performance to be decreased.

**TABLE I. QCG PARAMETERS USED IN THE EXAMPLE**

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>m</th>
<th>$\gamma = [\log_{2}^2+1]$</th>
<th>$N = m \cdot \gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>14</td>
<td>15</td>
<td>5</td>
<td>16</td>
<td>4</td>
<td>64</td>
</tr>
<tr>
<td>30</td>
<td>31</td>
<td>7</td>
<td>32</td>
<td>5</td>
<td>160</td>
</tr>
<tr>
<td>62</td>
<td>63</td>
<td>9</td>
<td>64</td>
<td>6</td>
<td>384</td>
</tr>
<tr>
<td>126</td>
<td>127</td>
<td>11</td>
<td>128</td>
<td>7</td>
<td>896</td>
</tr>
</tbody>
</table>

**Fig. 6.** Algorithm of SI detection in receiver.

V. EXAMPLE

In this section, to further explain the proposed scheme we assume an OFDM system with N subcarriers and 16-QAM modulation. Table I shows the QCG parameters used in the example for an OFDM system with $N=24,64,160,384$ and 896 subcarriers. The phases of $f_{i,n}$ with equal probabilities are set to either 0 or $\pi$. We suppose the flat Rayleigh fading channel for transmission and evaluate our scheme in terms of probability of SI detection failure (Pdf), PAPR reduction and bit error rate (BER).

A. Probability of SI detection failure (Pdf)

The probability of SI detection failure, Pdf, declares the probability that receiver cannot discover the side information correctly; that is, an entire received OFDM frame is lost. In Fig. 7, we have plotted the probability of SI detection failure Pdf vs. K for two values of the signal-to-noise ratio (SNR = 7 and 20dB) and the various numbers of OFDM subcarriers. From Fig. 7, it is clear that the SNR value
does not have a great affect on Pdf behavior. However, on the other hand, the values of \(N\) and \(K\) much influence Pdf performance. In higher value of constant \(K\), more distinction between expanded and non-expanded symbols is formed. Therefore, higher value of \(K\) causes the happening of a wrong detection event less likely. Additionally, whenever \(N\) rises the number of zero locations in each frame are also increased. So, it provides further valid estimates of the average energies in (11). Therefore, in our scheme, the more increase in \(N\), the better result can be got for any value of \(K\).

**B. BER performance evaluation**

It is also significant to engage in evaluation of bit error rate (BER) performance caused by the application of proposed scheme. In Fig. 8, we show BER vs. SNR values for our proposed scheme having supposed parameters in Table I with \(N=384\) subcarriers and various values of constant \(K\). Fig. 8 shows that the proposed technique can decrease the error performance. such reduction is as a result of energy rise of transmitted symbol. In other words, increasing the average energy of transmitted symbol has a great effect on BER performance. For comparison aim, the BER curve of an equivalent OFDM system employing conventional SLM with perfect side information, i.e. error-free SI, have also been shown in Fig. 8. The scenario of error-free SI is gained by employing the strong channel code absolutely devoted to the SI protection as a result of which, system complication and reduction of data rate will happen. As it is seen in Fig. 8, in BER=10\(^{-3}\) for \(K=1.5\) our proposed scheme lessens the error performance by only 0.5 dB in comparison with the conventional SLM with perfect SI. Additionally, increasing the \(K\) value make the gap of BER performance between our proposed scheme and error-free SI be decreased. When \(K=1.5\) is used this gap is very slight.

**C. Evaluation of PAPR reduction**

In Fig. 9, the complementary cumulative distribution function (CCDF) of the PAPR has been shown. We have considered performance of PAPR reduction, CCDF, with supposed parameters in Table I, \(K = 1.4\) as well as \(N=24,64,160\) and 384 subcarriers. For comparison aim, the PAPR curves obtained with the conventional SLM method and OFDM system without PAPR reduction have been plotted. These results have been achieved by application of oversampling=4 [23]. According to the results achieved in Fig. 9, the proposed method performance from the viewpoint of PAPR reduction is similar to that of the conventional SLM method.

**VI. CONCLUSION**

In this paper, we have proposed a new SLM scheme to reduce the PAPR without transmitting the explicit SI bits. In designing of our method, we have
used QCG algorithm which enables the phase coefficient vector to be random. Our examinations performed via considering an OFDM system by using 16-QAM modulation. It has been shown that the proposed scheme is specially interesting for OFDM systems employing a huge number of subcarriers. In reality, when the number of subcarriers and/or the expansion factor increase, the probability of SI detection failure decreases. Because the expanded and non-expanded symbols become more distinguishable from each other. When this probability is adequately low, it has been shown that the gap of BER performance between our proposed scheme and conventional SLM employing error-free side information become incomparable. Additionally, the proposed method performance from the viewpoint of PAPR reduction is similar to the conventional SLM method.

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