

# *Optimum Selection of Modes Set for Adaptive Modulation and Coding based on Imperfect Channel State Information*

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**Abstract**—Complexity issues and limited feedback rate strictly constrain the number of transmission modes in discrete link adaptation. A new algorithm is designed for selecting an optimum set of modes out of all the possible transmission modes based on the link's characteristics and the constraints, meanwhile, imperfect channel state information (CSI) is available. The design goal is to maximize the average spectral efficiency and the selection is done using a combination of nonlinear quantization and integer programming. As numerical evaluations for adaptive LDPC or convolutional coded, QAM modulated transmission show, performance of the proposed scheme is considerably improved in comparison with the benchmark schemes.

**Keywords**-component; Link Adaptation; Adaptive Modulation and Coding; Integer programming

## I. INTRODUCTION

Link adaptation is an efficient tool to conquest multipath fading in wireless communication systems where transmission rate is adapted based on the received SNR [1]. In practice, a limited set of modes may be utilized where each mode is implemented with a modulation and a coding scheme resulting in a specific transmission rate, i.e., adaptive modulation and coding (AMC). Obviously, the more the number of transmission modes are, the better the performance of the system would be. However, the implementation of each mode requires the use of appropriate hardware/software in the transmitter and the receiver while the hardware complexity and processing capability are limited especially in user end systems. In addition, another obstacle in which practical systems may be confront is that adapting rate based on perfect

channel state information (CSI) is impossible. That CSI is not existing because of error in channel estimation and time variant essence of channels.

Eventually, the index of appropriate mode must be sent back to the transmitter, as regards, increasing the number of modes would increase the feedback rate, so limited number of transmission modes must be considered.

AMC is implemented in IEEE 802.11 [2] and DVB [3] standards. Not only it is shown that the performance of different systems is improved by using AMC with perfect CSI [4]-[13], but also it can be observed that AMC increase system's performance with imperfect CSI [14]-[15]. In the mentioned articles, a limited number of transmission modes are selected randomly and the selected set remains unchanged, although, by the change of the link's characteristics or constraints. Based on the author's knowledge, no

method has been presented yet for optimum selection of modes set for AMC transmission.

In this article a new method is presented in which a set with  $M$  members is optimally selected from all the available transmission modes (a set with  $N$  modes) to maximize the average spectral efficiency (ASE) of the link. The selection is done based on the PDF of channel's SNR and the BER constraint. Design is done using a combination of nonlinear quantization and integer programming. At the beginning of the communication, modes set is selected and contracted between the receiver and the transmitter. After that, only the appropriate mode's index will send back to the transmitter. In this case, the required feedback rate is  $\lceil \log_2 M \rceil$ . In comparison with random selection of the modes or the case where the modes set is unchanged by the link's characteristics, numerical evaluations indicate that the performance is considerably improved. In the following, in section II, the scheme for optimum selection of modes set is presented. In section III, performance of the proposed scheme is examined using numerical evaluations. Benchmarks are set up as comparison basis to show the efficiency of the proposed algorithm. Finally the paper will be concluded in section IV.

II. OPTIMIZED SELECTION OF MODES SET

The goal is to design a scheme for selecting a subset with  $M$  members ( $\tilde{\mathcal{M}} = \{\tilde{R}_1 < \tilde{R}_2 < \dots < \tilde{R}_M\}$ ) out of all available transmission modes with  $N$  members ( $\mathcal{N} = \{\hat{R}_1 < \hat{R}_2 < \dots < \hat{R}_N\}$ ) to maximize ASE of the link. The instantaneous BER,  $pe(\tilde{\gamma}, k)$ , is constrained to  $B_0$ . If  $k$  and  $\tilde{\gamma}$ , respectively, show the instantaneous rate and perfect SNR of the link, the problem can be formulated as follows,

$$\tilde{\mathcal{M}} = \{\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_M\} = \arg \max_{\mathcal{M}=\{R_1, R_2, \dots, R_M\} \subset \mathcal{N}} ASE = E\{k\}, k \in \mathcal{M} \tag{1}$$

Subject to:

$$\begin{cases} C(1): \|\mathcal{M}\| = M \\ C(2): pe(\tilde{\gamma}, k) \leq B_0 \end{cases}$$

where  $\|\mathcal{M}\|$  represents the number of the members in the set  $\mathcal{M}$ . The above problem is a constrained discrete optimization problem that its solution is not straightforward. In the following, after description of channel model and imperfect CSI, at first 1, is reformulated based on the BER constraint. Next, an equivalent continuous problem is solved to obtain insight and compute the upper bound of performance. Finally, the solution of the reformulated problem will be presented.

A. Channel Model and Imperfect CSI

The communication system consists of a single input single output model over a Nakagami flat fading channel. As we supposed to available CSI is not perfect, so the estimated imperfect instantaneous SNR in the receiver would be  $\gamma$ . These two SNRs have a conditional distribution function with a correlation coefficient. Assuming a Nakagami flat fading channel of order  $\mu$ , the conditional distribution can be stated as follows [13],

$$f_{\tilde{\gamma}|\gamma} = \left(\frac{\tilde{\gamma}}{\rho\gamma}\right)^{\frac{\mu-1}{2}} \frac{1}{\Gamma} \exp\left(-\frac{\rho(\gamma+\tilde{\gamma})}{\Gamma(1-\rho)}\right) I_{\mu-1}\left(\frac{2}{\Gamma(1-\rho)}\sqrt{\rho\gamma\tilde{\gamma}}\right) \tag{2}$$

where  $\mu$  is Nakagami order,  $\Gamma$  represents the average of SNRs ( $\Gamma = E\{\gamma\} = E\{\tilde{\gamma}\}$ ),  $\rho = J_0^2(2\pi f_d T_d)$  is correlation between  $\gamma$  and  $\tilde{\gamma}$ ,  $J_0(\cdot)$  is Bessel function of the first type and zero order,  $f_d$  is the Doppler frequency and  $T_d$  is the round trip time of the channel and  $I_{\mu-1}(\cdot)$  shows the modified Bessel function of the first type and order of  $\mu - 1$ .

B. Problem Reformulation based on the BER constraint for imperfect CSI

In general, to guarantee the BER constraint with perfect CSI, i.e.,  $pe(\tilde{\gamma}, k) \leq B_0$ , it is required that  $\tilde{\gamma} \geq g_{B_0}(k)$  where  $g_{B_0}(\cdot)$  is a function of utilized coding and modulation scheme [14]. As it is discussed in [14] with imperfect CSI, we just may limit the probability of violation of BER from the required constraint, as follows,

$$Prob\{pe(\tilde{\gamma}, k) > B_0 | \gamma\} \leq B_v \tag{3}$$

However, just  $(1 - B_v)$  part of this transmission rate is reliable and is considered in the performance evaluation. To this end, it is required that,

$$g_{B_0}(k) \leq (Q_1\sqrt{\gamma} - Q_2)^2 \tag{4}$$

where  $Q_1 = \rho$ ,  $Q_2 = \sqrt{-(1-\rho)\ln(2B_v)}$  [14]. The  $\gamma_m$  in  $g_{B_0}(R_m) \leq (Q_1\sqrt{\gamma_m} - Q_2)^2$  is the minimum required SNR to satisfy the BER constraint when rate is  $R_m$ ,  $1 \leq m \leq M$ .

With modes set  $\mathcal{M} = \{R_1 < R_2 < \dots < R_M\}$ , and their corresponding thresholds as  $\gamma_1 < \gamma_2 < \dots < \gamma_M$ , ASE is computed as follows,

$$ASE = \sum_{m=1}^M g_{B_0}^{-1}\left((Q_1\sqrt{\gamma_m} - Q_2)^2\right) \int_{\gamma_m}^{\gamma_{m+1}} f_{\gamma}(\gamma) d\gamma \tag{5}$$

where  $f_{\gamma}(\cdot)$  is the probability density function (PDF) of imperfect SNR and  $g_{B_0}^{-1}(\cdot)$  is the inverse function of  $g_{B_0}(\cdot)$ .

There are  $N$  thresholds corresponding to  $N$  possible transmission rates as follows,

$$\hat{\Gamma} = \{\hat{\gamma}_1 < \dots < \hat{\gamma}_N\}, (Q_1\sqrt{\hat{\gamma}_n} - Q_2)^2 = g_{B_0}(\hat{R}_n), 1 \leq n \leq N; \hat{R}_n \in \mathcal{N} \tag{6}$$

Thus, 1 may be reformulated as,

$$\tilde{\Gamma} = \arg \max_{\Gamma=\{\gamma_1 < \gamma_2 < \dots < \gamma_M\} \subset \hat{\Gamma}} ASE(\gamma_1, \dots, \gamma_M) \tag{7}$$

where C(1) in 1 is now considered in the number of selected thresholds and C(2) is provisioned in the relation between the thresholds and the rates as in 4. Problem 7, is an unconstrained discrete optimization that may be solved by checking all the possible answers and select the best one. The number of possible subsets of  $\hat{\Gamma}$  with  $M$  members is  $\frac{N!}{M!(N-M)!}$ ; clearly, complexity of exhaustive search becomes too high when  $N$  is large.

In the next section, the equivalent continuous optimization problem is solved to obtain analytical insights and in the section after next, solution of 7 is presented.



### C. Performance analysis considering continuous form of the optimization problem

In this section, to obtain analytical insight we assume that the modes set in Problem 7 could contains any  $M$  real values and its members are not obliged to be selected from  $\hat{\Gamma}$  in 6. To solve the equivalent optimization problem. At first step, we need a closed form function for  $g_{B_0}(\cdot)$ , then we present the solution.

#### 1) A closed form function to estimate $g_{B_0}$

The function  $g_{B_0}(\cdot)$  may be computed by analytical performance analysis of the utilized coding and modulation schemes [16]. As the presented analytical bounds are not precise enough, we numerically evaluate the  $g_{B_0}(\hat{R}_n)$  values by simulation of the link's performance. Then, with the help of curve fitting, a continuous function is fit for computing  $g_{B_0}^{-1}(\tilde{\gamma})$  in a form of  $A \ln(B\tilde{\gamma} + C)$ , where  $A$ ,  $B$  and  $C$  are constants depending on  $B_0$ . The function  $g_{B_0}^{-1}(\tilde{\gamma})$  shows the maximum transmissible rate which satisfies the BER constraint. We compute  $g_{B_0}^{-1}(\tilde{\gamma})$  for different set of rates ( $\mathcal{N}$ ) with convolutional or LDPC coded QAM modulated modes. In Figures 1 and 2, the values of  $\hat{R}_n$  versus  $\hat{\gamma}_n$  and  $g_{B_0}^{-1}(\tilde{\gamma})$  are plotted for two selected sets. Evidently, the approximate functions are precise enough. Note that in this stage estimation of  $g_{B_0}(\cdot)$  is based on perfect CSI.

#### 2) Solution of the continuous form of the optimization problem

If rates weren't discrete and instantaneous transmission rate was continuously adapted versus SNR, ASE of continuous rate adaptation based on 5 and 4 would be,

$$ASE_{cont\_rate\_adp} = (1 - B_v) \int_{\max(\frac{1-C}{B}, 0)}^{\infty} A \ln(B(Q_1\sqrt{\gamma} - Q_2)^2 + C) f_{\gamma}(\gamma) d\gamma \quad 8$$

In 7, if just the number of transmission modes was limited, a candidate set for optimum values of the thresholds would be obtained by solving  $\vec{\nabla} ASE(\gamma_1, \dots, \gamma_M) = 0$ , where  $\vec{\nabla}$  denotes the gradient of ASE. This leads to the following system of equations with  $M$  equations  $M$  unknowns ( $1 \leq m \leq M$ ).

$$\begin{aligned} \vec{\nabla} ASE(\gamma_1, \dots, \gamma_M) = 0 &\Rightarrow \frac{\partial ASE}{\partial \gamma_m} = 0 \quad 9 \\ &\Rightarrow \frac{Q_1 - \frac{Q_2}{\sqrt{\gamma_m^*}}}{(Q_1\sqrt{\gamma_m^*} - Q_2)^2 + \frac{C}{B}} \int_{\gamma_m^*}^{\gamma_{m+1}^*} f_{\gamma}(\gamma) d\gamma \\ &= f_{\gamma}(\gamma_m^*) \ln \left( \frac{(Q_1\sqrt{\gamma_m^*} - Q_2)^2 + \frac{C}{B}}{(Q_1\sqrt{\gamma_{m-1}^*} - Q_2)^2 + \frac{C}{B}} \right) \\ \gamma_0^* &= \max\left(\frac{1-C}{B}, 0\right), \quad \gamma_{M+1}^* = \infty \end{aligned}$$

For a general Nakagami-m fading model, it could be shown that the hessian of  $ASE(\gamma_1, \dots, \gamma_M)$  with respect to the thresholds,  $-\nabla^2 ASE(\gamma_1, \dots, \gamma_M)$ , is positive semi-definite and thus  $ASE(\gamma_1, \dots, \gamma_M)$  is a

concave function of the thresholds (note to Appendix I).

According to the computed thresholds, ASE with optimized quantization is obtained as,

$$ASE_{opt\_quant} = (1 - B_v) \sum_{m=1}^M A \ln \left( B(Q_1\sqrt{\gamma_m^*} - Q_2)^2 + C \right) \int_{\gamma_m^*}^{\gamma_{m+1}^*} f_{\gamma}(\gamma) d\gamma \quad 10$$

This is a kind of purpose driven nonlinear quantization.

To show the efficiency of the mentioned quantization, we consider the most intuitive solution in which the range of  $\gamma$  is partitioned by the thresholds  $\gamma_1^u, \dots, \gamma_M^u$  into  $M$  intervals with the same probability of  $\frac{1}{M}$ . The ASE with equal probability quantization is labeled as  $ASE_{equ\_prob\_quant}$  and is computed as follows,

$$ASE_{equ\_prob\_quant} = (1 - B_v) \sum_{m=1}^M A \ln \left( B(Q_1\sqrt{\gamma_m^u} - Q_2)^2 + C \right) \int_{\gamma_m^u}^{\gamma_{m+1}^u} f_{\gamma}(\gamma) d\gamma \quad 11$$

In Fig. 3, the values of  $ASE_{equ\_prob\_quant}$  are compared with  $ASE_{opt\_quant}$  for a selected set. As shown a considerable outperformance by the purpose driven optimized quantization is achieved.

#### D. Solution of discrete optimization problem 7

Not only the number of transmission modes is limited, but also transmission rates are to be selected from  $\mathcal{N} = \{\hat{R}_1, \hat{R}_2, \dots, \hat{R}_N\}$  and thresholds are to be selected from  $\hat{\Gamma} = \{\hat{\gamma}_1, \hat{\gamma}_2, \dots, \hat{\gamma}_N\}$ . To solve 7, the results of 9 can be rounded to the closest values in  $\hat{\Gamma}$ . However, this might not necessarily lead to satisfaction of the problem's constraints and might be far from the optimum condition. All of these issues are considered by using integer programming methods like as the branch-and-bound method [17].

By the branch-and-bound method, optimum values of the thresholds are set from  $\tilde{\gamma}_M$  to  $\tilde{\gamma}_1$ . To set  $\tilde{\gamma}_M$ , at first, the system of equations in 9 is solved to find  $\{\gamma_1^*, \dots, \gamma_M^*\}$ . Next, two consecutive member in  $\hat{\Gamma}$  are found such that  $\hat{\gamma}_i < \gamma_M^* < \hat{\gamma}_{i+1}$ . Then, ASE is computed for the two set of thresholds,  $\{\gamma_1^*, \dots, \gamma_{M-1}^*, \hat{\gamma}_i\}$  and  $\{\gamma_1^*, \dots, \gamma_{M-1}^*, \hat{\gamma}_{i+1}\}$ . Finally,  $\tilde{\gamma}_M$  is set to each of  $\hat{\gamma}_i$  and  $\hat{\gamma}_{i+1}$  which leads to the more ASE. To set  $\tilde{\gamma}_{M-1}$ , the system of equations in 9 is resolved in which  $\gamma_M$  is fixed to  $\tilde{\gamma}_M$ . After finding  $\gamma_1^*, \dots, \gamma_{M-1}^*$ , two consecutive member in  $\hat{\Gamma}$  are found such that  $\hat{\gamma}_i < \gamma_{M-1}^* < \hat{\gamma}_{i+1}$ . Then,  $\tilde{\gamma}_{M-1}$  is set to each of  $\hat{\gamma}_i$  and  $\hat{\gamma}_{i+1}$  which leads to the more ASE. Step by step, the same approach is followed to set all of the thresholds.

The proposed algorithm to find the optimum set of thresholds ( $\tilde{\Gamma} = \{\tilde{\gamma}_1, \tilde{\gamma}_2, \dots, \tilde{\gamma}_M\}$ ) is summarized as follows:

- 1) Input: Initiate  $B_v$  and determine channel characteristics.
- 2) Solve system of equations in 9,



- $\vec{\gamma} ASE(\gamma_1, \gamma_2, \dots, \gamma_M) = 0$  and compute  $\{\gamma_1^*, \gamma_2^*, \dots, \gamma_M^*\}$ .
- 3) Find two consecutive members of  $\hat{F}$ ,  $\hat{\gamma}_{i+1}, \hat{\gamma}_i \in \hat{F}$  such that  $\hat{\gamma}_i < \gamma_M^* < \hat{\gamma}_{i+1}$ .
  - 4) If  $ASE(\gamma_1^*, \dots, \gamma_{M-1}^*, \hat{\gamma}_{i+1}) > ASE(\gamma_1^*, \dots, \gamma_{M-1}^*, \hat{\gamma}_i)$  then  $\tilde{\gamma}_M = \hat{\gamma}_{i+1}$  else  $\tilde{\gamma}_M = \hat{\gamma}_i$
  - 5) For  $k = 1: M - 1$ , do:
    - a. Resolve system of equations in 9 when  $\gamma_{M-k+1}, \dots, \gamma_M$  are fixed to  $\tilde{\gamma}_{M-k+1}, \dots, \tilde{\gamma}_M$  with  $M - k$  equations and  $M - k$  unknowns, i.e.,
 
$$\vec{\gamma} ASE(\gamma_1, \gamma_2, \dots, \gamma_{M-k}, \tilde{\gamma}_{M-k+1}, \dots, \tilde{\gamma}_M) = 0,$$
 and find newly values of  $\gamma_1^*, \gamma_2^*, \dots, \gamma_{M-k}^*$ .
    - b. Find two consecutive members of  $\hat{F}$ ,  $\hat{\gamma}_{i+1}, \hat{\gamma}_i \in \hat{F}$  such that  $\hat{\gamma}_i < \gamma_{M-k}^* \leq \hat{\gamma}_{i+1}$ .
    - c. If  $ASE(\gamma_1^*, \dots, \hat{\gamma}_{i+1}, \tilde{\gamma}_{M-k+1}, \dots, \tilde{\gamma}_M) > ASE(\gamma_1^*, \dots, \hat{\gamma}_i, \tilde{\gamma}_{M-k+1}, \dots, \tilde{\gamma}_M)$  then  $\tilde{\gamma}_{M-k} = \hat{\gamma}_{i+1}$  else  $\tilde{\gamma}_{M-k} = \hat{\gamma}_i$ .
  - 6) Output:  $\tilde{\Gamma} = \{\tilde{\gamma}_1, \tilde{\gamma}_2, \dots, \tilde{\gamma}_M\}$  and  $ASE_{BB} = ASE(\{\tilde{\gamma}_1, \tilde{\gamma}_2, \dots, \tilde{\gamma}_M\})$

### III. NUMERICAL EVALUATIONS

To evaluate the proposed scheme, two sets of transmission modes are considered. The first set is generated by various convolutional codes and QAM modulations. One of the best convolutional codes with the rate of  $R = 1/2$ , memory length of 6 and the code generator  $G = [133\ 171]_{oct}$  is used and making use of different puncturing patterns, twelve new codes with rates 1/2, 1/3, 3/4, 4/5, 5/6, 6/7, 7/8, 8/9, 9/10, 10/11, 11/12, 12/13, are produced [18]. Then, by using QAM modulation, BPSK-4QAM-8QAM-16QAM-32QAM-64QAM,  $N = 72$  transmission modes are created.

The second set includes  $N = 33$  modes whose coding and modulation schemes are extracted from the DVBS2 standard where a combination of LDPC and interleaving BCH codes are used with rates 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 5/6, 8/9, 9/10, and the modulations are 4QAM-16QAM and 64QAM [3].

Fig. 1 and Fig.2 show the performance of the first and the second set of modes. The values of  $\hat{R}_n$  are plotted versus  $g_{B_0}(\hat{R}_n)$ . As evident the fit function,  $g_{B_0}^{-1}(\gamma)$ , has an acceptable precision.

Fig. 3 is devoted to numerically evaluate the computations in section II-C-2; for different number of modes from DVBS2 standard the ASE of optimized quantization,  $ASE_{opt\_quant}$  is compared with ASE of continuous rate adaptation,  $ASE_{cont\_rate\_adp}$  in 8, and ASE when the range of SNR is quantized into equal probability intervals,  $ASE_{equ\_prob\_quant}$  in 11. As shown,  $ASE_{opt\_quant}$  has a considerable outperformance in comparison with  $ASE_{equ\_prob\_quant}$ . This shows the efficiency of purpose driven quantization.

To evaluate the efficiency of the proposed scheme for the selection of modes set, two benchmarks are setup for comparison. In the first one, the range of SNR is partitioned into  $M$  intervals such that probability of each interval is  $1/M$ . One threshold is selected from each interval. This is the most intuitive solution for the

mode selection. The resulting ASE by the mentioned scheme is named as  $ASE_{equ\_prob\_part}$ . The second scenario is considered to investigate the importance of selecting modes set based on the link's characteristics where the modes set which is optimized for the case with average SNR of 0 dB is used for all SNRs.

In Fig. 4, the first set of modes are used and the number of selected modes is  $||\mathcal{M}|| = 3$ . It is assumed that  $B_0 = 10^{-4}$  and order of Nakagami-m fading is  $\mu = 1$ . The normalized values of ASE (ratio of ASE to ASE of continuous rate adaptation) are plotted versus various average SNRs. Three scenarios are compared with each other; optimum mode selection, selection from equal probability intervals and using a fixed modes set (modes set which is optimized for average SNR of 0dB) for all average SNRs.

In Fig. 5, normalized ASE is plotted versus number of selected modes for different orders of Nakagami-m fading,  $\mu = 1$  or 3. The second set of modes are utilized with average SNR,  $E\{\gamma\} = -1dB$ . It is assumed that  $B_0 = 10^{-5}$ .

In Fig. 6,  $ASE_{BB}$  is plotted versus average SNR of the link for three different values of  $\rho$ . It is assumed that  $\mu = 1$  and the first set of modes are utilized with  $B_0 = 10^{-5}$ .

Noting figures 4, 5 and 6 the following noticeable observations are made:

- The ASE is considerably improved with the proposed optimum mode selection algorithm compared to the case in which modes are selected from equal probability intervals. The mentioned bench mark is the most intuitive scenario however with low average SNR, it may not be possible to find proper modes in the intervals, although with powerful LDPC codes.
- The ASE by the proposed scheme is considerably improved in comparison to the case in which modes set is fixed by changing the link's characteristics.

### IV. CONCLUSION

A new algorithm was designed for optimal selection of modes set for an adaptive modulation and coding transmission, while, imperfect channel state information is available. The selection was done based on the statistics of SNR and the BER constraint. The selected modes set is used after codifying an agreement between the receiver and the transmitter at the beginning of the communication. After determining the modes set, only the index of appropriate mode is computed and sent back to the transmitter. Numerical evaluations indicate that with the proposed algorithm the performance may be considerably improved in comparison with other benchmarks, meanwhile imperfect CSI does not cause to restrict this algorithm to find best set of modes.

### APPENDIX I

To proof that ASE in 8 is a concave function of the thresholds, it should be shown that determinate of Hessian matrix ( $\tilde{H}_{M \times M}$ ) always is non-positive. We have:



$$\tilde{H}_{M \times M} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1,M} \\ h_{21} & h_{22} & \dots & h_{2,M} \\ \dots & \dots & \dots & \dots \\ h_{M,1} & h_{M,2} & \dots & h_{M,M} \end{bmatrix}, h_{i,j} = \frac{\partial^2 ASE}{\partial \gamma_i \partial \gamma_j} \quad 12$$

where the elements of this matrix ( $h_{i,j}$ ) are obtained by 8 and 9 as follows,

$$h_{i,j} = \begin{cases} \frac{A f_{\gamma}(\gamma_j)}{\gamma_i + \frac{C}{B}} & i = j - 1 \\ \frac{A f_{\gamma}(\gamma_i)}{\gamma_j + \frac{C}{B}} & i = j + 1 \\ 0 & \text{other wise} \end{cases}, 1 \leq i, j \leq M \quad 13$$

It can be shown easily that  $|\tilde{H}_{M \times M}|$  is always non-positive.

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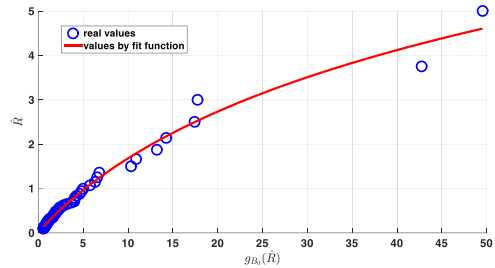


Fig. 1.  $\hat{R}_n$  versus  $g_{B_0}(\hat{R}_n)$ ,  $1 \leq n \leq N = 72$ ; for the first set of modes

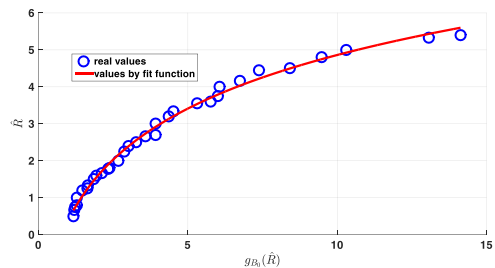


Fig. 2.  $\hat{R}_n$  versus  $g_{B_0}(\hat{R}_n)$ ,  $1 \leq n \leq N = 33$ ; for the second set of modes

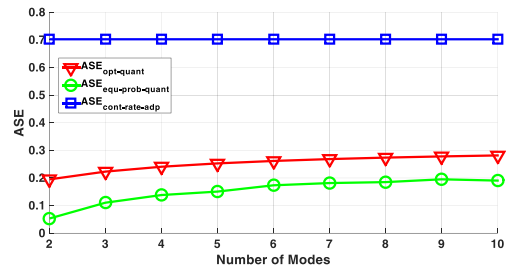


Fig. 3. Comparison of ASE versus number of modes for different scenarios (8, 10 and 11); The second set of modes are utilized, Nakagami-m fading model with  $\mu = 4$  and average SNR of 0dB;  $\rho = 0.95$  and  $B_0 = 10^{-5}$

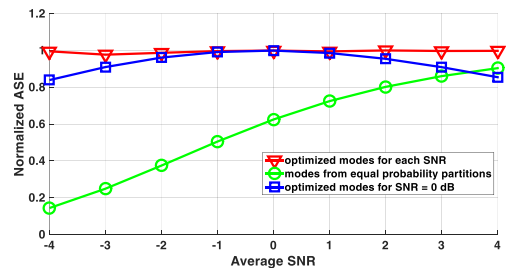


Fig. 4. Normalized ASE (ASE divided to  $ASE_{opt\_quant}$  in 11) versus the average SNR, comparison of different scenarios; first set of modes; Nakagami-m fading model with  $\mu = 1$ ;  $B_0 = 10^{-4}$ ;  $\rho = 0.95$  and number of modes  $M = 3$



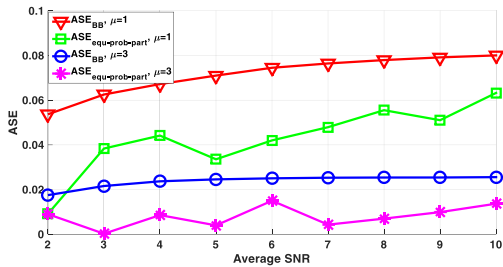


Fig. 5. ASE versus the number of modes for different order of Nakagami-m fading  $\mu = 1$  or 3; optimum mode selection in comparison with selection from equal probability intervals; first set of modes;  $\rho = 0.99$  and  $B_0 = 10^{-4}$ .

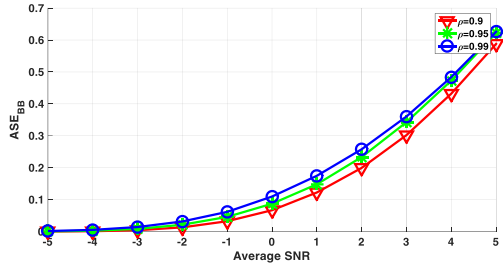


Fig. 6.  $ASE_{BB}$  for three different values of  $\rho$ ; first set of modes,  $\mu = 1$  and  $B_0 = 10^{-5}$



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