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A Stochastic Geometry Analysis of Hybrid Cellular-Adhoc CDMA Networks

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Abstract—**This paper studies the impact of using multihop wireless adhoc network in conjunction with cellular networks under a probabilistic routing strategy. Our objective is to obtain the outage probability of the two-dimensional hybrid CDMA network under path loss. In the proposed network the base stations are placed on a regular grid and users are distributed randomly according to a homogeneous Poisson point process. To obtain the outage probability of the proposed hybrid network, we study the outage probability of a cellular CDMA wireless network on both uplink and downlink as well as the outage probability of a multihop ad hoc CDMA network. We validate our analytical expressions via Monte Carlo simulations. Our results demonstrate that in order to improve the performance of a hybrid wireless network with a large density of users, it is beneficial to use multihop ad hoc communication.**

Keywords-hybrid networks; stochastic geometry;point process.

I. INTRODUCTION

Performance of cellular wireless networks has been mostly studied. In a two dimensional hexagonal cellular network, base stations are located at the center of a hexagonal cell and connected to a backbone wired network. Users communicate with each other via base stations and there is no direct or multihop communication between them. In comparison with cellular networks, ad hoc networks have no infrastructure. In these networks users communicate with each other directly or in a multihop fashion.

There have been many studies on the performance of ad hoc networks. Gupta and Kumar [1] analyzed the capacity of a wireless ad hoc network. n nodes are randomly distributed in the network and each node chooses its destination randomly. It is shown that the aggregate throughput capacity of all nodes for a random ad hoc network is $\Theta(\sqrt{\frac{n}{\log n}})$ and for an arbitrary ad hoc network is $\Theta(\sqrt{n})$. The throughput capacity of a three-dimensional random ad hoc network is also studied by these authors in [2]. It is shown that the aggregate throughput capacity is $\Theta((\frac{n}{\log n})^{\frac{2}{3}})$. In [3], the effect of mobility on throughput capacity of an ad hoc network where n nodes communicate with random source-destination pairs is studied. Using one hop relaying mechanism, an aggregate throughput capacity of $\Theta(n)$ is obtained.

In [4], nodes are distributed according to a homogeneous Poisson point process and the closed form probability distribution function for the aggregate interference at a receiver is obtained just for path loss exponent 4. In [5], the total capacity of a DS-CDMA random ad hoc network by using MMSE and MF receivers is investigated. It is shown that when MMSE detector is used, the ergodic capacity is higher than when MF is used. In [6], a new metric for ad hoc network capacity termed *Transmission Capacity* is defined. Stochastic geometry is used to show that FH-CDMA obtains a higher *Transmission Capacity* than DS-CDMA on the order of $M^{1-\frac{2}{\alpha}}$, where M and α are the spreading factor and path loss exponent, respectively. It is also used to show the effect of channel variation [7], interference cancellation [8] and scheduling [9] on the *Transmission Capacity* of ad hoc networks. In [10], the authors consider an interference-limited environment and free space path loss channel model as [6], and they derive a closed form expression for the outage capacity of an ad hoc wireless network. They show that an optimum threshold value exists that maximize the outage capacity.

In order to improve the spatial reuse and performance of cellular networks, multihop wireless ad hoc networks are used in conjunction with cellular networks. These kinds of networks are called cellular-ad hoc or hybrid networks. Limited numbers of studies have been done on different kinds of these networks. Most of these studies are focused on the two dimensional disk model proposed in [1]. In [11] a sparse network of base stations which are connected by a high-bandwidth wired network is placed in an ad hoc network. The results show that with n nodes and m base stations, the use of base station as relays is beneficial if m grows asymptotically faster than $(\sqrt{n})$.

In [12], the authors consider a hybrid network with just one active source and destination pair. The other $(n-2)$ nodes act as relays. Nodes are located uniformly in a disk of unit area. It is shown that the throughput capacity between source and destination scale as $\Theta(\log n)$. The capacity of three classes of wireless networks is studied in [13]. It is shown that for a hybrid wireless network with n nodes and n^d access points throughput gains on the order of n^d is achievable for $\frac{1}{2} < d < 1$. In [14], it is shown that by employing power control the $\Theta(n)$ per node throughput capacity can be achieved. [15] studied the benefits of using a hybrid network above the pure ad hoc network in terms of per node throughput capacity. It is shown that by adding an infrastructure the $\Theta(\sqrt{n/\log n})$ fold better performance than the pure ad hoc network can be obtained.

The capacity of both one and two dimensional hybrid networks with regular placement of base station and nodes is computed analytically in [16]. In the proposed hybrid network the coverage area of each base station is reduced and traffic is relayed by the users within the area to the users outside the area. It is shown that unlike the hexagonal hybrid network, linear hybrid network does not provide significant capacity improvement. In [17] and [18] a hybrid network similar to [16] is considered with the small number of hops and relays. In [17], a centralized downlink scheduling scheme in a cellular network is proposed and the throughput gain of the proposed hybrid network is evaluated with four relays. The upper bounds to throughput gains by considering geographical routing for uniform placement of relays are derived in [18]. In [19], the impact of the network dimensionality and geometry on the capacity of hybrid networks is investigated. They have shown that the dimensions of network lead to different capacity scaling laws. It is shown that for a one-dimensional network of n nodes and m base stations, the gain in capacity increases with the number of base stations. But for a two dimensional square network a large number of base stations is required.

Most of the above studies consider hybrid networks with deterministic placement of users. In this paper we consider a two dimensional hexagonal cellular CDMA network in which mobile users are distributed according to a homogeneous Poisson point process (PPP) with intensity λ. Since CDMA systems are interference limited, the interference generated by the users is a significant factor in determining the performance of these systems. We utilize the stochastic geometry approach to characterize the statistics of interference and evaluate the outage probability of the proposed hybrid network.

The organization of this paper can be summarized as follows: In Section II, we describe the components of our proposed hybrid network and determine the outage probability at a typical receiver. Section III, includes the simulations results and the comparison between analytical and simulation results. We conclude the paper in Section IV.

II. HYBRID NETWORK MODEL

The Hybrid network utilizes a combination of two topologies: 1) The ad hoc topology and 2) The infrastructure topology. In the infrastructure topology, users communicate with each other via base stations and in the ad hoc topology they exploit intermediate nodes as relays and data forwarded in a multihop fashion. We consider a hexagonal cellular CDMA network in which base stations are located at the center of hexagonal cells in fixed locations. They use omnidirectional antennas and are connected by a wired network. Mobile users are distributed according to a homogeneous PPP with intensity λ. Our proposed hybrid network employs a probabilistic routing strategy. In this model the transmitter selects one of the topologies according to some given probability.

We consider simple path loss channel model with the power law function $l(x) = x^{-\alpha}$, where $\alpha > 2$ is the path loss exponent. Therefore, the received power from a transmitter at distance d is given by

$$P_r = P_t K \left(\frac{d}{d_{ref}} \right)^{-\alpha},$$

where d_{ref} is a reference distance, and K depends on the antenna characteristics. This model only valid at transmission distances $d > d_{ref}$.

We have three types of traffic in this network: uplink, downlink and ad hoc. In order to refuse the interference between traffics, we separate them in the frequency domain. Our goal is to determine the outage probability at a typical receiver.

According to our proposed hybrid network, the outage probability at a typical receiver is a conditional probability which is defined as:

$$\Pr\{\text{outage}\} = \Pr\{\text{outage}|\, I\}\Pr\{I\} + \Pr\{\text{outage}|\, A\}\Pr\{A\}, \quad (1)$$

where $\Pr\{I\}$ is the probability of selecting infrastructure topology for transmission and $\Pr\{A\}$ is the probability of selecting ad hoc topology.

The outage probability due to selecting the infrastructure topology is the combination of two events:

$$\Pr\{\text{outage} \,|\, I\} = \Pr\{D\} \cup \Pr\{U\}, \quad (2)$$

where $\Pr\{D\}$ is the downlink outage probability of infrastructure topology, and $\Pr\{U\}$ is the uplink outage probability of this topology.

In the following, we will analytically compute the outage probabilities for ad hoc, uplink and downlink modes.

A. The outage probability in infrastructure topology

As we saw in (2), the outage probability of infrastructure topology is the combination of two outage probabilities 1) downlink outage probability and 2) uplink outage probability. We continue to calculate these probabilities separately.

1) Downlink outage probability: Let $\Pi = \{X_i, i \geq 1\}$ be a homogeneous PPP on the plane $\mathbb{R}^2$ with intensity λ. Therefore the mean number of users per cell is $N = \lambda A$, where A is the cell site area. We assume that the probability of being a node in downlink mode is p_d. Thus from the independent thinning property of PPP, the set of users in this mode forms a thinned Poisson point process with intensity $p_d \lambda$ which we denoted by Π_d [20]. We consider user X_i at cell site k. The distance between user X_i^k and it's serving base station bs_k is denoted by D_i^k. The Signal to Interference-plus-Noise Ratio (SINR) for this user is given by

$$\gamma_i^k = \frac{p_i^k g_i^k}{\eta(I_{intracell,i}^k - p_i^k g_i^k) + I_{intercell,i}^k + N_0},$$

where N_0 is the power of noise, $g_i^k = (D_i^k)^{-\alpha}$ and p_i^k are the path gain and transmit power from base station bs_k to the user X_i^k, respectively. η is the orthogonality factor, which quantifies the loss of orthogonality between CDMA codes and varies from 0 to 1. $\eta = 1$ means that the codes are perfectly non-orthogonal and $\eta = 0$ means that interferences are neglected. Assuming $N_0 = 0$, p_i^k is given by:

$$p_i^k = \frac{\gamma_i^k}{1+\eta\gamma_i^k}(I_{intracell,i}^k / g_i^k)(\eta + I_{intercell,i}^k / I_{intracell,i}^k).$$

Inter-cell interference ($I_{intercell,i}^k$) is caused by the downlink transmitted powers of all base stations around cell k. Intra-cell interference ($I_{intracell,i}^k$) is due to the signals that base station bs_k transmits to the other users of this cell, if we denote total power of base station k by P^k then $I_{intracell,i}^k = g_i^k P^k$.

Because of the homogeneity of network, we assume that the total power of all base stations is the same. We define the ratio $f_i^k = I_{intercell,i}^k / I_{intracell,i}^k$ as the interference factor. Therefore, the transmitted power for user X_i^k is given by:

$$p_i^k = \frac{\gamma_i^k}{1+\eta\gamma_i^k}(\eta + f_i^k)P^k. \quad (3)$$

The total power transmitted by base station k is

$$P^k = P^c + \sum_i p_i^k,$$

where P^c represents the transmit power of all downlink common channels which is a fraction of maximal power of base station $P^c = \varepsilon P_{max}$. Assuming perfect power control, all users have the same SINR $\gamma_i^k = \gamma$. According to (3)

$$P^k = \frac{P^c}{1 - \frac{\gamma}{1+\eta\gamma}\sum_i(\eta + f_i^k)}. \quad (4)$$

Outage occurs at cell site k if the total power of base station P^k exceeds the maximal transmitting power, thus we deduce from (4):

$$\begin{aligned}\Pr\{\text{outage} \,|\, D\} &= \Pr\{P^k > P_{max}\} \\ &= \Pr\{\sum_i(\eta + f_i^k) > \frac{1-\varepsilon}{\beta}\},\end{aligned} \quad (5)$$

where $\beta = \frac{\gamma}{1+\eta\gamma}$.

We define random variable Y^k as $Y^k = \sum_i(\eta + f_i^k)$. In order to calculate the outage probability, first we assume constant f_i^k as [21], [22], and then we consider f_i^k as a function of the distance to the base station D_i^k [23].

a) Analysis I: Assuming constant f_i^k, Y^k is a Poisson process with parameter N_d and the outage probability is given by:

$$\Pr\{\text{outage} \,|\, D\} = \sum_{j=\varphi}^{\infty} \frac{e^{-N_d}(N_d)^j}{j!}, \quad (6)$$

where $N_d = \lambda A p_d$ is the mean number of users per cell in down link mode and

$$\varphi = \left\lceil \frac{1-\varepsilon}{(\eta+f)\beta} \right\rceil,$$

where $\lceil z \rceil$ is the smallest integer greater than or equal to the z.

b) Analysis II: Let us mark the points of Π_d with their related interference factor f. Thus, $\Pi_d^* = \{X_i, f_i\}$ is a marked Poisson point process with intensity $p_d \lambda$ [18]. We define the interference factor f_i^k of user X_i^k as follows

$$f_i^k \triangleq \frac{2\pi\rho_{BS}(D_i^k)^\alpha}{(\alpha-2)(2R_c - D_i^k)^{\alpha-2}},$$

which is obtained in [23] as a function of the distance to the base station for a large network. R_c is the radius of cell sites, $\rho_{BS} = \frac{1}{\pi R_c^2}$ is the density of base stations per m^2.

Thus, Y^k can be separated into two terms

$$Y^k = \sum_i \eta + \sum_i f_i^k = Y_a^k + Y_f^k,$$

where Y_a^k is a Poisson random variable with mean ηN_d. Therefore, the downlink outage probability of cell site k is given by

$$\Pr\{\text{outage} \mid D\} = \Pr\{Y^k > \upsilon \mid Y_a^k > 0\}$$
$$= \frac{e^{-\eta N_d}}{1-e^{-\eta N_d}} \sum_{j=1}^{\infty} \frac{(-\eta N_d)^j}{j!} \Pr\{Y_f^k > \upsilon - (j-1)\}$$

where $\upsilon = \frac{1-\varepsilon}{\beta}$.

In order to calculate the outage probability, we consider a Gaussian approximation for Y_f^k according to the central limit theorem. Therefore, the downlink outage probability is given by

$$\Pr\{\text{outage} \mid D\} = \frac{e^{-\eta N_d}}{1-e^{-\eta N_d}} \sum_{j=1}^{\infty} \frac{(-\eta N_d)^j}{j!} \times Q\left(\frac{\upsilon - j + 1 - \mu_d}{\sqrt{\sigma_d^2}}\right), \quad (7)$$

where Q is the "Q-function" for the standard normal distribution, μ_d and σ_d^2 are the mean and variance of Y_f^k, respectively. According to the homogeneous distribution of mobile users the probability density of D_i^k is given by:

$$f_D(d) = \frac{2d}{R_c^2} \quad d \le R_c.$$

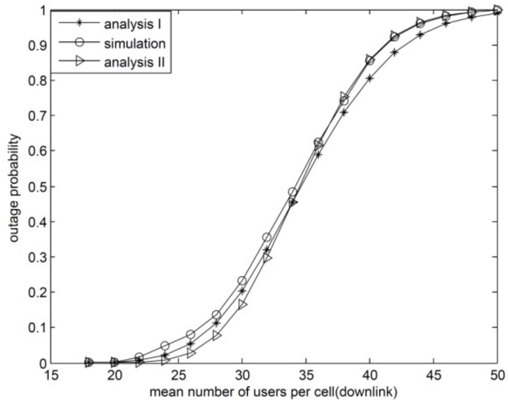

Fig 1. Downlink outage probability vs. the mean number of users per cell

Therefore, by using Campbell's theorem [20] the mean and variance of Y_f^k are obtained as follows:

$$E(Y_f^k) = \int_0^{R_c} p_d \lambda 2\pi r dr \int_0^{R_c} \frac{2\pi\rho_{BS}}{(\alpha-2)} \frac{x^\alpha}{(2R_c - x)^{\alpha-2}} \frac{2x}{R_c^2} dx$$
$$= \frac{4 p_d \lambda \pi R_c^2}{2^{\alpha-2}(\alpha-2)(\alpha+2)} {}_2F_1(\alpha-2, \alpha+2, \alpha+3, \tfrac{1}{2})$$

$$Var(Y_f^k) = p_d \lambda 2\pi r dr \int_0^{R_c} \frac{2\pi\rho_{BS}}{(\alpha-2)} \frac{x^{2\alpha}}{(2R_c - x)^{2\alpha-4}} \frac{2x}{R_c^2} dx$$
$$= \frac{2\pi p_d \lambda R_c^4}{2^{2\alpha-4}(\alpha-2)(\alpha+1)} {}_2F_1(2\alpha-4, 2\alpha+2, 2\alpha+3, \tfrac{1}{2}),$$

where ${}_2F_1(a,b,c,t)$ is the hypergeometric function and its integral form is given by:

$${}_2F_1(a,b,c,t) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(b-c)} \int_0^1 \frac{x^{b-1}(1-x)^{c-b-1}}{(1-xt)^a} dx,$$

and $\Gamma(.)$ is the gamma function.

In Fig.1 we validate our numerical results by Monte Carlo simulation for path-loss exponent $\alpha = 4$. This figure shows the downlink outage probability versus the mean number of users per cell for both analyses and simulation. In the first analysis the interference factor is constant and assumed to be $f = 0.4$.

2) Uplink outage probability: In this section, we evaluate the outage probability in uplink mode under the same path-loss model. We assume that each node transmits in uplink mode with probability p_u. Therefore, the set of transmitters in this mode is a thinned version of the Poisson point process Π with intensity $p_u \lambda$. We denote this process by Π_u.

We investigate the uplink outage probability of the center cell site. We use non-orthogonal CDMA codes due to difficulty of users synchronization, thus the orthogonality factor equals to 1. According to our channel model each transmitter is power controlled by the nearest base station. We employ the perfect power control (all users require the same SINR level) to compensate the near-far effect. By assuming perfect power control, the transmitted signals from the mobile users of the same cell site are received at the same power level at the base station. With these assumptions, outage occurs for all uplink transmitters of center cell simultaneously if the total interference received at the base station exceeds a given threshold.

The total interference received at the base station is the combination of intra-cell interference and inter-cell interference $I^u_{total} = I^u_{intracell} + I^u_{intercell}$. Intra-cell interference is created by the uplink transmitters in the center cell site and inter-cell interference is imposed by the uplink transmitters in other cell sites. For the transmitters in the center cell site the transmitted signals are received in unit power, thus the $I^u_{intracell}$ is a Poisson random variable with parameter N_u. $N_u = p_u \lambda A$ is the mean number of users per cell in uplink mode. The outage probability at the center cell site is given by

$$\Pr\{\text{outage} \mid U\} = \Pr\{I^u_{total} > \delta \mid I^u_{intracell} > 0\} = \frac{e^{-N_u}}{1-e^{-N_u}} \sum_{j=1}^{\infty} \frac{(N_u)^j}{j!} \Pr\{I^u_{intercell} > \nu\}, \quad (8)$$

where $\nu = \delta - j + 1$.

To obtain the outage probability of uplink mode, we need to characterize the statistics of inter-cell interference. We mark every point $\{X_i\}$ in Π_u by its distance $\{D_i\}$ to the nearest base station. Therefore, $\Pi^*_u = \{X_i, D_i\}$ is a homogeneous marked PPP with intensity $p_u \lambda$.

Consequently, the inter-cell interference created by users located in other cell sites is obtained as

$$I^u_{intercell} = \sum_{(X_i, D_i) \in \Pi^*_u} \left(\frac{D_i}{|X_i|} \right)^{\alpha} 1(X_i \in \bar{b}(O, R_c)),$$

where $|X_i|$ denotes the distance between node i and the origin and

$$1(x \in A) = \begin{cases} 1 & \text{for } x \in A \\ 0 & \text{otherwise} \end{cases},$$

is the indicator function.

According to the Central Limit Theorem we apply a Gaussian approximation to the inter-cell interference as [24], [25], [26] and [27]. We use the mean and variance of $I^u_{intercell}$ as the mean and variance of the Gaussian approximation. Considering these assumptions

$$\Pr\{I^u_{intercell} > \nu\} = Q\left(\frac{\nu - \mu_u}{\sqrt{\sigma^2_u}} \right), \quad (9)$$

where μ_u and σ^2_u are the mean and variance of $I^u_{intercell}$, respectively. Using Campbell's Theorem [20] μ_u and σ^2_u are obtained as follows:

$$\mathrm{E}[I^u_{intercell}] = 2\pi p_u \lambda \int_{R_c}^{\infty} r^{-\alpha} r \mathrm{d}r \int_0^{R_c} \frac{2x}{R_c^2} x^{\alpha} \mathrm{d}x = \frac{4\pi p_u \lambda R_c^2}{(\alpha-2)(\alpha+2)},$$

$$Var[I^u_{intercell}] = 2\pi p_u \lambda \int_{R_c}^{\infty} r^{-2\alpha} r \mathrm{d}r \int_0^{R_c} \frac{2x}{R_c^2} x^{2\alpha} \mathrm{d}x = \frac{\pi p_u \lambda R_c^2}{(\alpha-1)(\alpha+1)}.$$

Substituting (8) in (7), we obtain the following approximation to the uplink outage probability at the center cell site:

$$\Pr\{\text{outage} \mid U\} = \frac{e^{-N_u}}{1-e^{-N_u}} \sum_{j=1}^{\infty} \frac{(N_u)^j}{j!} Q\left(\frac{\nu - \mu_u}{\sqrt{\sigma^2_u}} \right). \quad (10)$$

We plot the analytical and Monte Carlo simulation results of uplink outage probability for $\alpha = 4$ in Fig. 2.

B. The outage probability in ad hoc topology

In this section, we calculate the outage probability of ad hoc topology. We assume that transmitters select this topology with probability p_a under the same channel model. Thus from the (independent) thinning property of PPP, the set of transmitters in this mode forms a PPP of intensity $p_a \lambda$ [20], which denoted by Π_a. We also consider that the receiver is capable to cancel the interference of interfering nodes located at distance $r < 1$. We apply the power control mechanism proposed in [6], which is called as *pairwise power control*. In this mechanism the transmitter chooses its transmission power such that the received power at its intended receiver will be some fixed level p. Thus according to our channel model the transmission power will be pd^{α} for $d > 1$ and the path-loss exponent $\alpha > 2$.

We utilize the furthest neighbor routing proposed in [28] and represented in Fig. 3 to balance the energy consumption and decrease delay. We consider that each transmitter chooses its furthest neighbor within the given distance d as receiver.

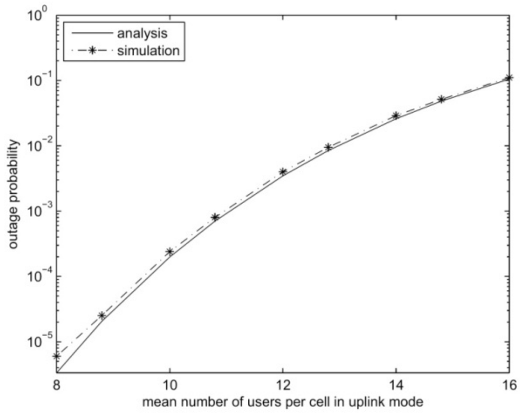

Fig 2. Uplink outage probability vs. the mean number of users per cell.

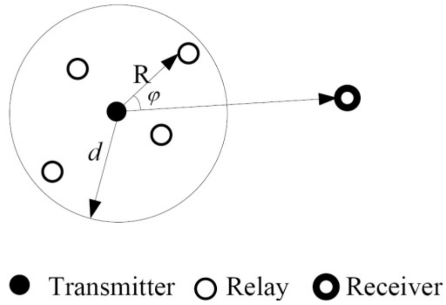

Fig 3. Furthest neighbor routing scheme.

The probability distribution function of the distance to the furthest neighbor within distance d for a two dimensional network is given by

$$f_R(r) = \frac{2\lambda\phi r e^{\lambda\phi r^2}}{e^{\lambda\phi d^2}-1} \qquad r \in [1,d],$$

where $0 < \phi \le \frac{\pi}{2}$. For efficient routing the angle between the vector to the next hop and source-destination vector must be smaller than ϕ .

We mark the transmitters in Π_a with the distance to their furthest neighbors. With these assumptions $\Pi_a^* = \{X_i, R_i\}$ is a marked homogeneous PPP, where X_i is the location of transmitter i in ad hoc mode and R_i is the distance between the transmitter and its intended receiver.

In order to obtain the outage probability, we condition on a typical receiver at the origin which we are interested in calculating the outage probability. It results from the Slyvniak's theorem [20] that the distribution of the point process is not change by adding a receiver at the origin. The outage probability for the receiver at the origin is

$$\Pr\{\text{outage} \mid A\} = \Pr\{\frac{p}{N_0 + \sum_{(X_i,R_i)\in\Pi_a^*} p(\frac{R_i}{|X_i|})^{\alpha}} \le \beta_a\},$$

where N_0 is the power of the thermal noise, β_a is the SINR required for successful reception and $|X_i|$ indicates the distance from node $i \in \Pi_a^*$ to the origin.

We define the random variable I_a as the aggregate interference at the receiver. Thus, the outage occurs at the receiver if

$$I_a = \sum_{(X_i,R_i)\in\Pi_a^*} \left(\frac{R_i}{|X_i|}\right)^{\alpha} \ge \theta,$$

where $\theta = \frac{1}{\beta_a} - \frac{N_0}{p}$.

To calculate the outage probability, we need to characterize the statistics of I_a . We approximate the distribution of random variable I_a with a log-normal random variable as [27]. The mean and variance of the log-normal random variable are the mean and variance of I_a .

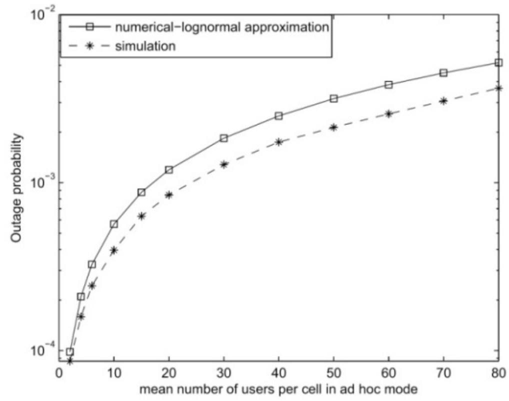

Fig 4. Outage probability of ad hoc CDMA network vs. the mean number of users in ad hoc mode.

Using this approximation the outage probability is given by:

$$\Pr\{\text{outage} \mid A\} = \Pr\{I_a \ge \theta\} \qquad (11)$$
$$= Q\left(\frac{\ln(\theta) + 5\ln(\sigma_a^2 + \mu_a^2) - 2\ln\mu_a}{\sqrt{\ln(\sigma_a^2 + \mu_a^2) - 2\ln\mu_a}}\right),$$

where μ_a and σ_a^2 are the mean and variance of I_a , respectively.

Applying Campbell's Theorem [20] the mean and variance of I_a are given by:

$$\mathrm{E}(I_a) = 2p_a\pi\lambda\int_1^{\infty} r^{-\alpha} r dr \int_1^d \frac{2\lambda\phi x^{\alpha+1} e^{\lambda\phi x^2}}{e^{\lambda\phi d^2}-1}dx$$
$$= \frac{2\pi p_a \lambda^{1-\alpha/2}\phi^{-\alpha/2}(-1)^{\alpha/2}}{(\alpha-2)(e^{\lambda\phi d^2}-1)}$$
$$\times[\gamma(\frac{\alpha+2}{2}, -\lambda\phi) - \gamma(\frac{\alpha+2}{2}, -\lambda\phi d^2)],$$

$$Var(I_a) = 2p_a\pi\lambda\int_1^{\infty} r^{-2\alpha} r dr \int_1^d \frac{2\lambda\phi x^{2\alpha+1} e^{\lambda\phi x^2}}{e^{\lambda\phi d^2}-1}dx$$
$$= \frac{p_a\pi\lambda^{1-\alpha}\phi^{-\alpha}(-1)^{\alpha}}{(\alpha-1)(e^{\lambda\phi d^2}-1)}$$
$$\times\left[\gamma(\alpha+1, -\lambda\phi) - \gamma(\alpha+1, -\lambda\phi d^2)\right],$$

where $\gamma(s,x)$ is the lower incomplete gamma function, whose integral form is given by:

$$\gamma(s,x) = \int_0^x t^{s-1} e^{-t} dt.$$

Fig. 4 presents the analytical results as well as Monte Carlo simulation for the outage probability in ad hoc topology for $\alpha = 4$ and $\phi = \frac{\pi}{2}$.

III. Simulation and Numerical Results

In this section the simulation results are compared to the obtained analytical derivations for the outage probability of proposed hybrid network. We consider a cellular CDMA network which contains 37 hexagonal cells. Base stations are located at the center of cells with omnidirectional antennas. We set the radius of each cell to 1 Km. Mobile users are distributed according to a homogeneous Poisson point process over space. The voice activity is about $\upsilon = 0.4$ and the path loss exponent is assumed to be $\alpha = 4$. Fig. 5

shows the simulation and numerical results for the outage probability of the hybrid network at a typical receiver. The probability that a mobile user transmits in ad hoc mode is set to $p_a = 0.75$.

Fig. 6 shows the analytical and simulation results for the outage probability of hybrid network versus the probability of transmitting in ad hoc topology p_a for $N = 100$. It can be observed from this figure that the outage probability of the hybrid network is minimized for large values of p_a. Fig. 7 shows the numerical results versus mean number of users for various values of p_a. Based on these figures it can be concluded that for dense hybrid networks, since adjacent users are close to each other, it is beneficial to employ ad hoc topology for transmission. Fig. 7 shows that employing the infrastructure topology does not improve considerably the outage probability of hybrid network for low density of users.

IV. CONCLUTION

The outage probability of a hybrid cellular-ad hoc CDMA network under a probabilistic routing strategy is studied. In the proposed network the transmitter selects one of the infrastructure or ad hoc modes with some probability. In our hexagonal hybrid network the base stations are positioned at the center of cell sites and are connected to each other. Users are distributed according to a homogeneous Poisson point process with intensity λ. By using stochastic geometry tools, closed form expressions for the outage probability of a CDMA cellular network (in both uplink and downlink modes) and the outage probability of a CDMA ad hoc network are obtained. These results are validated via Monte Carlo simulation separately, and are used to obtain the outage probability of hybrid network. We showed that considerable improvement in outage probability is not achieved by using infrastructure topology in low user density environment. But it can be observed that for the large density of users, it is beneficial to use ad hoc mode for transmission.

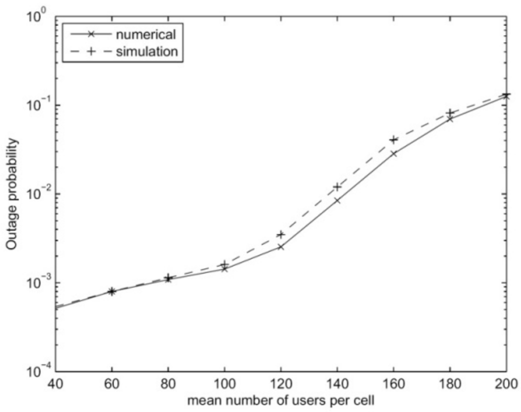

Fig 5. Outage probability of hybrid CDMA network vs. the mean number of users.

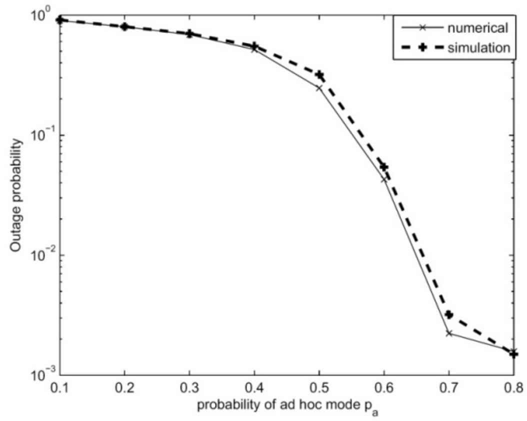

Fig 6. Outage probability of hybrid network versus the probability of transmitting in ad hoc topology.

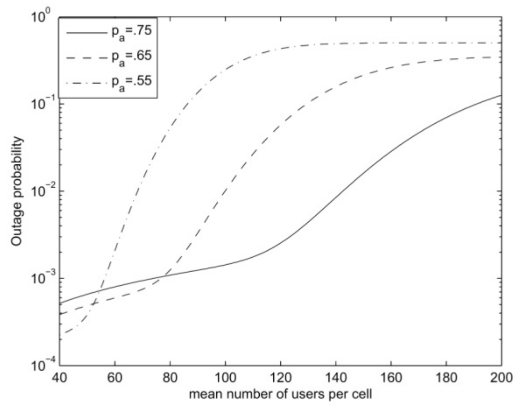

Fig 7. Outage probability of hybrid network versus the mean number of users per cell for various values of p_a.

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