Bayesian-Based Cooperative Framework for Spectrum Sensing in Cognitive Radio Networks

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Abstract—**Energy detection has been adopted as an alternative spectrum sensing method for cognitive radios due to its low computational complexity and not requiring a priori information about the signal to be detected. However, noise uncertainty and hidden terminal problem make energy detector practically challenging specially in low signal-to-noise ratio (SNR) regime. Collaboration among multiple cognitive radios has been recognized as a practical strategy to improve the reliability of spectrum sensing. In this paper, a cooperative spectrum sensing framework is proposed to blindly determine the occupancy of a wideband spectrum. Specifically, contrary to conventional energy detector, the proposed method does not require any knowledge of noise variance to detect the presence of primary signals. Moreover, diversity achieved by cooperation enables the framework to maintain a reasonable performance even in low SNR values. Simulation results confirm the effectiveness of our proposed method in improving both the probabilities of detection and false alarm.**

Keywords- cognitive radio, cooperative spectrum sensing, energy detection, Generalized Bayesian estimator

I. INTRODUCTION

The increasing demand for radio spectrum has resulted in overcrowding the utilized bands and the need for higher data rates. Considering the limitations of the natural frequency spectrum, it becomes obvious that the current strategy of static frequency allocation schemes cannot accommodate the requirements of high data rate devices. Consequently, innovative techniques are needed to efficiently utilize the available spectrum. Cognitive Radio (CR) arises to be a tempting solution to the spectral congestion problem by introducing opportunistic usage of the frequency bands that are not heavily occupied by licensed users [1-3]. As a matter of fact, recent measurements by Federal Communications Commission (FCC) have shown that 70% of the allocated spectrum in US is not utilized [2].

It is well known that the realization of the spectrum access largely depends on finding the spectrum holes, i.e. spectrum sensing [4, 5]. As a result, sensing accuracy has been considered as the most significant factor to evaluate the performance of CR networks. Hence, recent researches have been focused on improving the sensing accuracy considering the practical constraints. Generally, spectrum sensing techniques can be classified into three main categories: energy detection [6, 7], matched filter detection [8], and cyclostationary feature detection [9]. If the secondary user (SU) has no a

priori information about the features of the primary signals except local noise statistics, then the energy detector is the optimal method [10]. When some knowledge about the primary user (PU) signal such as pilots, preambles, or synchronization messages are known to the SUs, the matched filter is usually the optimal detector [11]. On the other hand, the cyclostationary feature detectors can differentiate the primary signal from the interference and local noise by exploiting certain periodicity existed in the received signal if the modulation schemes of the primary signals are known. This advantage is gained at the expense of more computational complexity [12]. In this paper, we will adopt the energy detector as the building block for the proposed spectrum sensing technique due to its advantages from implementation viewpoint.

There are several factors which make energy detector practically challenging: i) the performance of energy detector highly depends on noise power which cannot be accurately estimated [13], ii) destructive channel conditions between the PUs and the SUs can greatly degrade the detection performance [10-13].

Several papers have studied the role of noise uncertainty on the performance degradation of energy detector and proposed methods to overcome this problem. As a case in point, [14] has shown that under additive white Gaussian noise (AWGN) assumption, the variance-based methods have better performance than energy detection with noise uncertainty. In addition, Taherpour et al. in [15] proposed a wideband spectrum sensing scheme based on generalized likelihood ratio detector under noise uncertainty assumption. Furthermore, in [16], a two-stage algorithm has been introduced which is robust against noise uncertainty.

However, these schemes cannot mitigate the multi-path fading and hidden terminal problem. In order to improve the reliability of spectrum sensing, multiple secondary users can collaborate to conduct spectrum sensing and take advantage of spatial diversity. It has been shown that cooperative spectrum sensing techniques can alleviate the problem of corrupted detection [10-13, 17]. The authors in [18] proposed a cooperative blind combination technique based on the principle of maximizing the SNR. The algorithm does not need any prior knowledge of the primary and noise signal, but it is required to estimate the covariance matrix of the received signals. In addition, [19] performs a blind spectrum sensing based on empirical characteristic function of the observed sample vectors. However, these methods may encounter severe degradation when the utilized approximations (covariance matrix or samples' characteristic function) are not reasonably precise.

Motivated from all the above and as an alternative approach, the aim of this paper is to utilize the Bayesian estimation introduced in [16] to propose a cooperative spectrum sensing framework without the utilization of any approximation. Specifically, we have proposed Generalized Bayesian Estimation Energy Detection (GBEED) technique in which multiple SUs collaboratively determine the presence or the absence of PUs in a wideband spectrum. To eliminate the dependency of the conventional energy detector to the noise variance, we utilize the non-informative prior

probability distribution [20]. Thus uncertainty in noise variance estimation does not affect the performance of
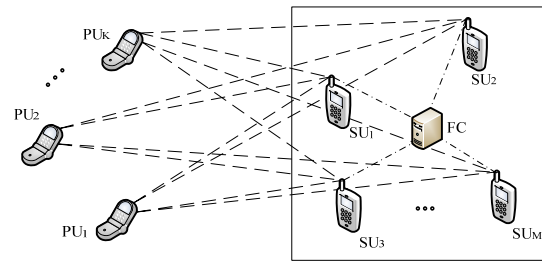

Fig 1. Scheme of SUs, FC and PU location.

the derived algorithm. Simulation results confirm the effectiveness of GBEED algorithm in the fading environment and the presence of noise uncertainty even in low SNR regime.

Notation: Throughout the paper, we use uppercase boldface letters to represent matrices and lowercase bold letters to denote vectors. $(.)^T$ stands for transpose of a vector. In addition, $\mathbb{N}(\mu, \sigma^2)$ and $\mathbb{CN}(\mu, \sigma^2)$, respectively, denote a real and complex Gaussian distribution with mean μ and variance σ^2. Also, $|.|$ represents the magnitude of a complex number and $E\{.\}$ stands for statistical expectation. Finally, $det(.)$ denotes the determinant of a matrix and $\propto$ and $\approx$ are the proportional and approximation symbols.

The rest of the paper is organized as follows. In section II, the system model and the assumptions are given. Section III is devoted to the cooperative spectrum sensing framework. In the first subsection, the conventional energy detector is overviewed and the rest of the section is dedicated to the derivation of the proposed GBEED algorithm. Simulation results are presented in Section IV. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL AND ASSUMPTIONS

A. System Model

Let us consider a CR network with M SUs randomly located in a $d \times d$ area. We assume that the primary network consists of K distinct bands with the same bandwidth, i.g. W. The objective is to cooperatively estimate the number of occupied channels and determine their locations.

Consider $x_{mk}(n)$ as the n'th sample of the k'th band, received by m'th SU; hence, $x_{mk}(n)$ can be written as

$$x_{mk}(n) = h_{mk}\theta_k(n) + w_{mk}(n),$$

$$m = 1, \dots, M, \quad k = 1, \dots, K \qquad (1)$$

where $w_{mk} \sim \mathbb{CN}(0, \sigma^2)$ is the noise component and h_{mk} denotes the channel gain between the m'th SU and the PU occupies the k'th band. Herein θ_k represents the absence (H_0) or presence (H_1) of the PU in the k'th channel. Therefore, the hypothesis testing problem for θ_k can be formulated as follows

$$\theta_k(n) = \begin{cases} 0, & H_0 \\ s_k(n), & H_1 \end{cases} \qquad (2)$$

where s_k is the primary signal in the k'th band.

In our system model, the local power measurements from all collaborating SUs are collected at a central unit, called fusion center (FC), which may be either one of the secondary nodes, or a separate control node. Then, the final decision about the hypothesis testing problem is made by the FC based on the received local measurements. In particular, by the utilization of GBEED algorithm, FC firstly estimates the number of occupied bands and then determines their location within all the bands.

B. Assumptions

The following assumptions will be retained for illustrative purpose and for analytical tractability:

1. It is assumed that the fading process remains static during each detection interval but it varies from one interval to another, and all channel elements are independent and identically distributed (i.i.d) random variables as $h_{mk} \sim \mathbb{CN}(0, \sigma_{mk}^2)$.
2. SUs calculate the power of their observations in each band and forward them to the FC over an error-free channel and in an orthogonal manner, for example by the use of TDMA protocol.
3. FC utilized the proposed GBEED algorithm to blindly make the final decision about the occupied channels.
4. We assume that there is no available knowledge about the noise variance and the sensing channel gains parameter.

III. COOPERATIVE SPECTRUM SENSING FRAMEWORK

A. Conventional Energy Detection

In a conventional energy detector, the power of the received signal of the k'th channel at the m'th SU is calculated according to

$$P_{mk} = \frac{1}{N} \sum_{n=1}^{N} |x_{mk}(n)|^2 \qquad (3)$$

Without loss of generality, we assume that $E\{|s_k|^2\} = 1$ [10], then for the large number of samples, by the use of central limit theorem, P_{mk} can be approximated as [21]

$$\begin{cases} H_0: & P_{mk} \sim \mathbb{N}(\sigma^2, \sigma^4/N) \\ H_1: & P_{mk} \sim \mathbb{N}(\delta_{mk}^2, \delta_{mk}^4/N) \end{cases} \qquad (4)$$

where σ^2 represents the noise variance at the SU's, $\delta_{mk}^2 = \sigma_{mk}^2 + \sigma^2$ and σ_{mk}^2 denotes the variance of channel gain between the m'th SU and the PU which occupies the k'th band.

The calculated powers are then transmitted to the FC to make the final decision. Considering equal gain combiner, we have the following average power for each band

$$P_k = \frac{1}{M} \sum_{m=1}^{M} P_{mk}, \qquad k = 1,2,\dots,K \qquad (5)$$

Without loss of generality, we assume that the vector $\boldsymbol{P} = [P_1, P_2, \dots, P_K]^T$ is in decreasing order. Finally, the hypothesis testing problem at the FC can be formulated as

$$\begin{cases} H_0: & P_k \sim \mathbb{N}\left(\sigma^2, \dfrac{\sigma^4}{MN}\right) \\ H_1: & P_k \sim \mathbb{N}\left(\dfrac{1}{M}\sum_{m=1}^{M}\delta_{mk}^2, \dfrac{1}{M^2 N}\sum_{m=1}^{M}\delta_{mk}^4\right) \end{cases} \qquad (6)$$

By considering the optimal likelihood ratio test (LRT) with a test threshold τ_k, we have

$$P_k \underset{H_0}{\overset{H_1}{\gtrless}} \tau_k \qquad (7)$$

Probability of detection (P_d) and probability of false alarm (P_f) are reliability and efficiency factors used to evaluate the sensing performance. P_d denotes the probability that a channel is sensed to be occupied when it is actually occupied and P_f is the probability that a channel is sensed to be occupied when it is actually idle. By the above definitions, P_d and P_f can be calculated as follows [21].

$$P_{d_k} = Q\left(\frac{\tau_k - (\sum_{m=1}^{M}\delta_{mk}^2)/M}{\sqrt{\sum_{m=1}^{M}\delta_{mk}^4}/(M\sqrt{N})}\right) \qquad (8\text{-}a)$$

$$P_{f_k} = Q\left(\frac{\tau_k - \sigma^2}{\sigma^2/\sqrt{MN}}\right) \qquad (8\text{-}b)$$

where $Q(.)$ is the *complementary cumulative distribution* function which calculates the tail probability of a zero mean unit variance Gaussian variable, i.e.

$$Q(z) = \int_{z}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

Assuming a target probability of false alarm $P_{f_k} = \beta$, and substituting it into (8-b), the test threshold can be calculated as [11]

$$\tau_k = \left(1 + Q^{-1}(\beta)/\sqrt{MN}\right)\sigma^2 \qquad (9)$$

It is obvious that sensing performance of the secondary network largely depends on the noise variance estimation. Thus the energy detection methods degrade significantly by existence of noise uncertainty. In the following subsection, we will propose the GBEED algorithm to combat noise uncertainty as well as fading effects and hidden terminal problem.

B. Problem formulation

In this paper, our objective is to find the number of occupied bands and their locations. Let $\boldsymbol{x}_m(n) = [x_{m1}(n), x_{m2}(n), \dots, x_{mK}(n)]^T$ denotes the observation vector at the m'th cooperating SU. The conditional probability of number of occupied bands, i.e. K', given the observation set $\boldsymbol{R}_m = [\boldsymbol{x}_m(1), \boldsymbol{x}_m(2), \dots, \boldsymbol{x}_m(N)]$ from all the SUs can be expressed as

$$\mathscr{p}(K'|\boldsymbol{R}_1, \boldsymbol{R}_2, \dots, \boldsymbol{R}_M)$$
$$= \frac{\mathscr{p}(\boldsymbol{R}_1, \boldsymbol{R}_2, \dots, \boldsymbol{R}_M|K')\mathscr{p}(K')}{\mathscr{p}(\boldsymbol{R}_1, \boldsymbol{R}_2, \dots, \boldsymbol{R}_M)} \qquad (10)$$

The probability $\mathscr{p}(\boldsymbol{R}_1, \boldsymbol{R}_2, \dots, \boldsymbol{R}_M)$ is the same for all possible values of K' and $\mathscr{p}(K')$ can be assumed non-

informative [16]. Hence, maximizing $\wp(K'|\boldsymbol{R}_1, \boldsymbol{R}_2, ..., \boldsymbol{R}_M)$ is equivalent to maximize $\wp(\boldsymbol{R}_1, \boldsymbol{R}_2, ..., \boldsymbol{R}_M|K')$.

Let $\psi^{K'} = (\delta_{11}, \delta_{12}, ..., \delta_{1K'}, \delta_{21}, ..., \delta_{MK'}, \sigma)$ denotes the set of unknown parameters. Then conditional probability of $(\boldsymbol{R}_1, \boldsymbol{R}_2, ..., \boldsymbol{R}_M)$ can be expressed as

$$\wp(\boldsymbol{R}_1, \boldsymbol{R}_2, ..., \boldsymbol{R}_M|K')$$
$$= \int_{\psi^{K'}} \wp(\boldsymbol{R}_1, \boldsymbol{R}_2, ..., \boldsymbol{R}_M|\psi^{K'}, K')\wp(\psi^{K'}|K')\, d\psi^{K'} \quad (11)$$

Calculating the above integral over the set of unknown parameters $\psi^{K'}$, allows us to eliminate the need for exact parameter set estimation. In the following subsection, our aim is to introduce a model for the prior distribution, $\wp(\psi^{K'}|K')$, hence calculating the integral in (11). Finally, we will propose the GBEED algorithm for the cooperative spectrum sensing.

C. Non-informative prior probability distribution

In Bayesian statistical inference, a prior probability distribution of an uncertain quantity ρ is the probability distribution that would express one's uncertainty about ρ before the data is taken into account. It is meant to attribute uncertainty rather than randomness to the uncertain quantity. Generally, priors are two types: informative and non-informative [20]. The informative prior refers to the case when there is a specific information about the variable, for example, noise component without uncertainty in variance has an informative prior probability since we describe it as a complex zero mean Gaussian variable with the known variance. On the other hand, non-informative prior expresses vague or general information about a variable. Since we have no knowledge of the parameter set, $\psi^{K'}$, we should choose a non-informative prior distribution for $\psi^{K'}$.

Among different proposed non-informative prior distributions, the Jeffreys' prior rule [22] is adopted for positive parameter like $\psi^{K'}$ which is defined as

$$\wp(\psi^{K'}|K') \propto \sqrt{det\left(\mathfrak{I}(\psi^{K'}|K')\right)} \quad (12)$$

where $\mathfrak{I}(.)$ denotes the fisher information [22]. Since the elements of the vector $\psi^{K'}$ are independent variables, we have

$$\wp(\psi^{K'}|K') = \wp(\sigma)\prod_{m=1}^{M}\prod_{k=1}^{K'}\wp(\delta_{mk}) \quad (13)$$

Since the received signal for different bands at the SUs are modeled as zero mean Gaussian variable, under hypothesis H_0, x_{mk} can be expressed as

$$\wp(x_{mk}|\sigma) \sim \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{|x_{mk}|^2}{2\sigma^2}} \quad (14)$$

Then, the Jeffreys prior for the standard deviation σ is

$$\wp(\sigma) \propto \sqrt{\mathfrak{I}(\sigma)}$$
$$= \sqrt{E\left\{\left[\frac{d}{d\sigma}log\big(\wp(x_{mk}|\sigma)\big)\right]^2\right\}}$$

$$= \sqrt{E\left\{\left[\frac{|x_{mk}|^2 - \sigma^2}{\sigma^3}\right]^2\right\}} \quad (15)$$

By the definition of statistical expectation, we obtain

$$\wp(\sigma)$$
$$\propto \sqrt{\int_{-\infty}^{+\infty}\wp(x_{mk}|\sigma)\left[\frac{|x_{mk}|^2 - \sigma^2}{\sigma^3}\right]^2 dx_{mk}}$$

$$= \sqrt{\frac{2}{\sigma^2}} \propto \frac{1}{\sigma} \quad (16)$$

With the same calculations, we have

$$\wp(\delta_{mk}) \propto \frac{1}{\delta_{mk}} \quad (17)$$

D. Generalized Bayesian Estimation Energy Detection (GBEED)

To calculate the integral in (11), we must find the conditional distribution of observations, $\boldsymbol{R}_1, \boldsymbol{R}_2, ..., \boldsymbol{R}_M$. In a wideband spectrum, it is logical to assume that signals in different bands are independent. Then, according to the assumption (1) and by considering the independency of primary signal samples, we have

$$\wp(\boldsymbol{R}_1, \boldsymbol{R}_2, ..., \boldsymbol{R}_M|\psi^{K'}, K')$$
$$= \prod_{m=1}^{M}\wp(\boldsymbol{R}_m|\psi^{K'}, K')$$
$$= \prod_{m=1}^{M}\prod_{n=1}^{N}\wp(x_m(n)|\psi^{K'}, K') \quad (18)$$

Consequently, we obtain

$$\wp(\boldsymbol{R}_1, \boldsymbol{R}_2, ..., \boldsymbol{R}_M|\psi^{K'}, K')$$
$$= \left(\frac{1}{\pi^{MK}(\sigma^2)^{M(K-K')}\prod_{m=1}^{M}\prod_{k=1}^{K'}\delta_{mk}^2}\right)^N$$
$$exp\left(-N\sum_{m=1}^{M}\sum_{k=1}^{K'}\frac{P_{mk}}{\delta_{mk}^2} - \frac{N}{\sigma^2}\sum_{m=1}^{M}\sum_{k=K'+1}^{K}P_{mk}\right) \quad (19)$$

Then by the use of (16), (17) and (19), the integral in (11) can be written as (20) on the top of the following page. To calculate (20), we use the subsequent equation [24]

$$\int_0^{\infty} z^{-(2a+1)}exp(-bz^{-2})\, dz = \frac{1}{2}\Gamma(a)b^{-a} \quad (21)$$

where $\Gamma(.)$ represents the gamma function and for integer values is defined as $\Gamma(a) = (a-1) \times (a-2) \times ... \times 1$. Therefore, we obtain (22-a) and (22-b) on the top of the following page. Finally, by the use of these equations, the integral in (11) is calculated as (23) on the top of the following page.

To further simplify the retained equation in (23), we use $\Gamma(z) \cong (2\pi)^{1/2}z^{z-(1/2)}exp(-z)$ as an

$$\mathscr{p}\left(\boldsymbol{R}_1, \boldsymbol{R}_2, \dots, \boldsymbol{R}_M \middle| \psi^{K'}, K'\right) \propto \int \left(\frac{1}{\pi^{MK}(\sigma^2)^{M(K-K')} \prod_{m=1}^{M} \prod_{k=1}^{K'} \delta_{mk}^2}\right)^N$$

$$exp\left(-N \sum_{m=1}^{M} \sum_{k=1}^{K'} \frac{P_{mk}}{\delta_{mk}^2} - \frac{N}{\sigma^2} \sum_{m=1}^{M} \sum_{k=K'+1}^{K} P_{mk}\right) \frac{1}{\sigma} \prod_{m=1}^{M} \prod_{k=1}^{K'} \frac{1}{\delta_{mk}} d\delta_{11}, d\delta_{12}, \dots, d\delta_{1K'}, d\delta_{21}, \dots, d\delta_{MK'}, d\sigma \quad (20)$$

$$\int_0^\infty \delta_{mk}^{-(2N+1)} exp(-NP_{mk}\delta_{mk}^{-2}) \, d\delta_{mk} = \frac{1}{2}\Gamma(N)(NP_{mk})^{-N} \quad (22\text{-}a)$$

$$\int_0^\infty \sigma^{-(2MN(K-K')+1)} exp\left(-N\sigma^{-2} \sum_{m=1}^{M} \sum_{k=K'+1}^{K} P_{mk}\right) d\sigma$$

$$= \frac{1}{2}\Gamma\big(MN(K-K')\big)\left(N \sum_{m=1}^{M} \sum_{k=K'+1}^{K} P_{mk}\right)^{-MN(K-K')} \quad (22\text{-}b)$$

$$\mathscr{p}\left(\boldsymbol{R}_1, \boldsymbol{R}_2, \dots, \boldsymbol{R}_M \middle| \psi^{K'}, K'\right) \propto \frac{1}{2^{MK'+1}} \frac{1}{\pi^{MKN}} \big(\Gamma(N)\big)^{MK'} \Gamma\big(MN(K-K')\big)$$

$$\left[\prod_{m=1}^{M} \prod_{k=1}^{K'} (NP_{mk})^{-N}\right]\left(N \sum_{m=1}^{M} \sum_{k=K'+1}^{K} P_{mk}\right)^{-MN(K-K')} \quad (23)$$

$$\frac{-\log f\left(\boldsymbol{R}_1, \boldsymbol{R}_2, \dots, \boldsymbol{R}_M \middle| \psi^{K'}, K'\right)}{N}$$

$$\propto \sum_{m=1}^{M} \sum_{k=1}^{K'} \log(P_{mk}) + M(K-K') \log\left(\sum_{m=1}^{M} \sum_{k=K'+1}^{K} P_{mk}\right)$$

$$- M(K-K') \log\big(M(K-K')\big) - \frac{MK'}{2N} \log\left(\frac{\pi}{2N}\right) + \frac{1}{2N} \log(K-K') \quad (25)$$

approximation for gamma function. Then, the following two equations are derived

$$\big(\Gamma(N)\big)^{MK'}$$
$$= (2\pi)^{MK'/2} N^{(N-(1/2))MK'} exp(-NMK') \quad (24\text{-}a)$$
$$\Gamma\big(MN(K-K')\big)$$
$$= (2\pi)^{1/2} \big(MN(K-K')\big)^{MN(K-K')-(1/2)} \quad (24\text{-}b)$$
$$exp\big(-MN(K-K')\big)$$

Using the above equations, the negative logarithmic transformation of (23) is expressed as (25) on the top of the this page.

Eventually, the proposed GBEED expression is given by

GBEED(K')

$$= \sum_{m=1}^{M} \sum_{k=1}^{K'} \log(P_{mk}) + M(K-K') \quad (26)$$

$$\log\left(\sum_{m=1}^{M} \sum_{k=K'+1}^{K} P_{mk}\right) + C(K', M)$$

where

$$C(K', M) = \frac{1}{2N}\log(K-K')$$

$$- M(K-K')\log\big(M(K-K')\big) - \frac{MK'}{2N}\log\left(\frac{\pi}{2N}\right)$$

And therefore the number of occupied bands can be estimated as

$$\widehat{K}' = arg \min_{K'=0,\dots,K-1} GBEED(K') \quad (27)$$

where the "minimum operator" is used for the negative logarithmic scale applied in (25). Since for $K' = K$, the equation for $C(K', M)$ has singularity, we cannot apply the algorithm for this case; However, the possibility of such occurrence is very low, for example $< (0.3)^{100} \approx 0$ for $K = 10$. Then, the bands correspond to the $\widehat{K}'$ largest values (first $\widehat{K}'$ elements) of the vector $\boldsymbol{P}$ are considered to be occupied. Hence,

the proposed GBEED algorithm is summarized in Table I.

TABLE I: THE PROPOSED GBEED ALGORITHM

1. The collaborating SUs individually calculate the power of their local observations for different bands (P_{mk}) and forward them to the FC.

2. The power vector $\boldsymbol{P}$ with the elements defined in (5) are calculated and arranged in decreasing order.

3. FC estimates the number of occupied bands based on

$$\widehat{K}' = arg \min_{K'=0,...,K-1} GBEED(K')$$

where $GBEED(K')$ is defined in (26).

4. The bands correspond to the $\widehat{K}'$ largest values of the vector $\boldsymbol{P}$ are considered to be occupied.

IV. SIMULATION RESULTS

To evaluate the performance of the GBEED algorithm, different sets of simulations are presented. We assume that the primary network consists of $K = 40$ bands. To generate different channel variances, we use the model $\sigma_{mk}^2 = \sigma_{tot}^2 + \alpha$ where α is a random variables uniformly selected in interval [-5, 5] in dB. Thus we define the average SNR as $SNR_{ave} = \sigma_{tot}^2/\sigma^2$. Furthermore, the global probabilities of detection and false alarm are calculated as averages over different bands and for 10^6 Monte Carlo runs.

In Figure 2, the normalized GBEED value versus different values of K' are depicted for different numbers of collaborative SUs and sample numbers and $SNR = -10dB$. In this figure, the correct number of occupied channels is assumed to be 15. As it is obvious from this figure, the minimum value of GBEED(K') is occurred at $K' = 15$ which confirms the right decision. Moreover, increasing the number of samples and cooperative nodes, results in a sharper curve around the minimum point and thus a higher accurate estimation of occupied channels.

The performance of the GBEED algorithm is compared with energy detection (ED) in the presence of noise uncertainty (NU) in Figure 3. In particular, we have depicted the average probability of miss-detection ($P_{md} = 1 - P_d$) and P_f for different bands versus SNR, i.e.

$$P_{md} = \frac{1}{K}\sum_{k=1}^{K} P_{md_k}, \quad P_f = \frac{1}{K}\sum_{k=1}^{K} P_{f_k}$$

The estimated noise variance in dB is modeled as $\hat{\sigma}^2 = \sigma^2 + \varepsilon$ where ε corresponds to uncertainty component and is uniformly generated as a random variable in the interval [-1, 1]. The threshold of ED is set to achieve $P_f = 10^{-3}$; however, its performance is degraded severely by noise uncertainty $(P_f \approx 0.5)$. On the other hand, noise uncertainty does not affect the performance of the proposed GBEED algorithm. Hence, in the presence of noise uncertainty, the GBEED algorithm outperforms the conventional ED, enormously. Considering accurate noise variance estimation, ED has better performance in terms of

P_{md} while in terms of P_f, GBEED algorithm acquires higher performance. Notice that ED requires the exact
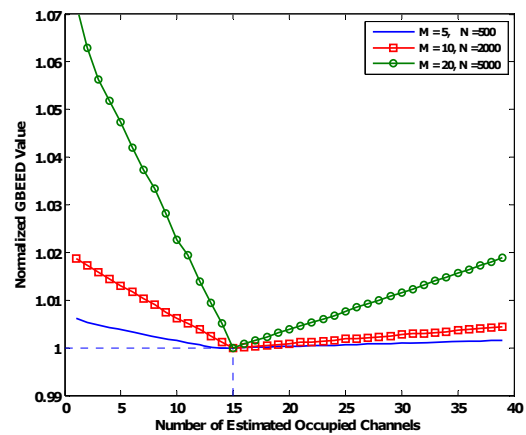

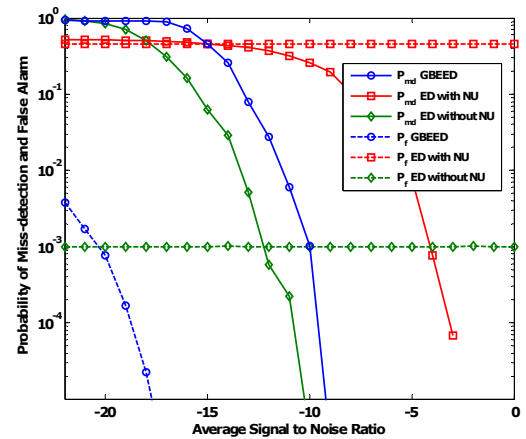Fig 2. Normalized GBEED value vs. K' for average   SNR= -10dB

Fig 3. Probabilities of miss-detection and false alarm vs. SNR for GBEED algorithm and conventional ED in presence and absence of noise uncertainty.
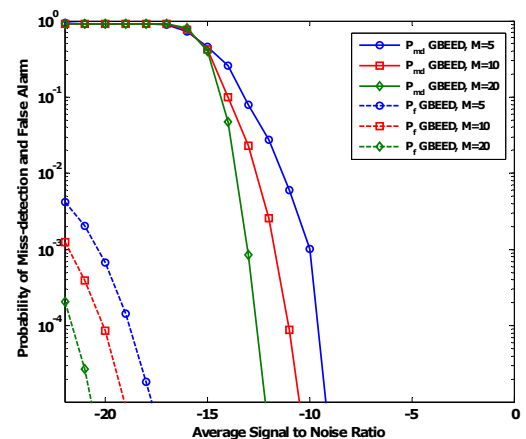

Fig 4. Probabilities of miss-detection and false alarm vs. SNR for $N = 5000$ and different number of SUs

estimation of noise variance to achieve higher probability of detection which is not applicable in practical scenarios. Probabilities of P_{md} and P_f versus SNR are illustrated in Figure 4 for different numbers of SUs and $N = 5000$. For multiple SUs, it is unlikely

that all of them experience a deep fading, simultaneously. Therefore, as it is expected, increasing
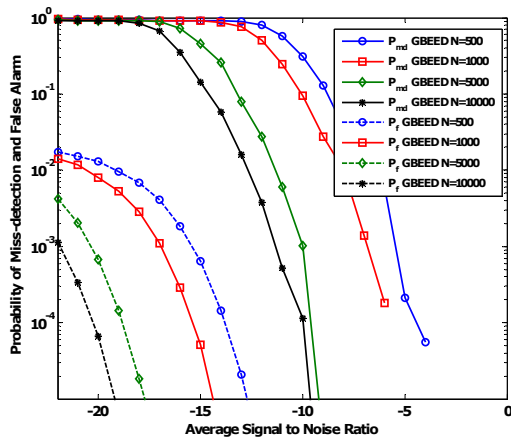

Fig 5. Probabilities of miss-detection and false alarm vs. SNR for $M = 5$ and different sample numbers.

the number of cooperating nodes improves the sensing performance in terms of both P_{md} and P_f.

Finally, the performance of GBEED algorithm is investigated for different sample numbers and $M = 5$. It can be seen that the performance is greatly improves as the sample number increases. This is due to the fact that according to (5), as $N \to \infty$, the variance under both hypotheses tends to zero. Thus the difference between H_0 and H_1 is perfectly distinguishable.

V.   CONCLUSIONS

In this paper, we proposed a cooperative sensing framework to determine the presence or absence of PUs in different bands. Contrary to energy detector, our method does not depend on noise variance estimation. Thus the noise uncertainty does not affect the performance of the sensing procedure. Moreover, cooperation among SUs combats the destructive fading conditions; therefore, further increase in detection performance is achieved Simulation results confirm the effectiveness of the proposed framework.

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