A Method to Improve Accuracy of Learner Model based on Knowledge and Belief of Learner

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Abstract—The learner model represents essential information about characteristics of learner. The Adaptive Educational Systems and Intelligent Tutoring Systems use learner model to adapt required learning services according to characteristics of each learner. Hence, the accuracy of learner model is an important issue. A learner model is called “open” if its parameters could be inspected, discussed or changed by users. In this paper a novel method is proposed to improve accuracy of learner model based on learner knowledge and learner belief about his/her model. For this purpose the overlay learner modeling with Bayesian networks is used to represent learner knowledge. Then according to nature of open learner model, the learner model is presented as skill meter and learner could state his/her belief about it. Then the model is updated through proposed method. Finally the method is evaluated by use of a comprehensive test and t-student test. The results show our method improves accuracy of learner model.

Keywords: Learner Model, Accuracy of Learner Model, Open Learner Model

I. INTRODUCTION

Adaptive Educational Systems (AES) and Intelligent Tutoring Systems (ITS) are interesting research domains in e-learning. Utilization of learner model in these systems is the most important characteristic which makes them different from other types of learning environments. In fact, learner model provides necessary information which capable the learning system to adapt its services to learning needs according to his/her knowledge, background, or other characteristics during the learning process [1, 2].

Open learner models are learner models that can be viewed or accessed in some way by the learner, or by other users (e.g. teachers, peers, parents) [3, 4].

According to the role of learner model in adaptation, the accuracy of learner model is an important issue. In open learner modeling, learner’s belief about his/her model could be used as a reference to update learner model [5, 6, 7]. Updating
the learner model in this way doesn’t always increase the accuracy of the learner model. In Open Learner Modeling personal learner models appropriately; this shows why there should be a mechanism to increase the accuracy in Open Learner Modeling.

In this paper a novel method is proposed to improve the accuracy of learner model based on knowledge and belief in Open Learner Modeling. For this purpose, first the learner knowledge is modeled with overlay learner modeling based on Bayesian networks. Then the model is presented as skill meter and learner could state his/her belief about it. Afterwards the model is updated through proposed method to increase its accuracy. At last the method is evaluated by comprehensive test and t-student test. The results show our method improves the accuracy of learner model.

This paper is structured in four sections. Present section which is an introduction on the work. The background theories are explained in the second section. In section three research methodology and results analysis are described. Finally, section four expresses conclusions and new trends of this work.

II. BACKGROUND THEORIES

A. Learner modeling

As explained in [8] the user modeling is traced back to the works of Allen, Cohen and Perrault (1978, 1979) and Elaine Rich (1989). Learner modeling is the term being selected in e-learning domain.

In [1] Brusilovsky mentioned that a Learner model includes parameters like: knowledge, interests, goals, are necessary: domain knowledge model and overlay knowledge model.

![Overlay User Model](image)

![Domain Model](image)

**Fig. 1. Overlay knowledge modeling**

**Domain knowledge model:** In this part the general knowledge of domain is divided into a set of domain knowledge fragments which are named as concepts. These concepts are connected to each other, thus some inter-concept inferring is possible. The connection between concepts could be in two forms:

a. A tree of educational objectives, where general objectives are progressively decomposed into specific objectives

b. Concepts can be connected by different kinds of relationships such as: *is-a, has, cause-effect.*

**Overlay knowledge model:** Overlay knowledge model is way of assigning value to each fragment of domain knowledge. Bayesian network is a common

Modeling learners may not have adequate knowledge to comment about their own educational backgrounds, emotional behaviors, and learning style.

The most important parameter of learner model is knowledge. The learner's knowledge is a dynamic parameter that changes from one session to another session, or even during the same session. Learner's knowledge will increase if the learner learns a new concept, but it may be considered a decline in knowledge if the learner forgets something. A simple and explicit way to obtain the learner's knowledge is taking a quiz. The rate of the correct answers to the quiz is considered as the level of the learner’s knowledge in a specific domain.

Modeling techniques such as: scalar [9], overlay [1, 9], perturbation [10] and stereotype [11] can be utilized to model the learner's knowledge.

B. Overlay Modeling

Overlay modeling approach is a dominant approach for learner modeling in ITS and AES. Overlay approach matches properly to the core function of AHS, and provides personalized access to information. Therefore, it has been accepted as de-facto standard by almost all educational and many non-educational adaptive hypermedia systems [1]. So in this work overlay knowledge modeling has been used. The idea of overlay knowledge modeling is to represent the knowledge of each individual user as a subset of a domain knowledge model [12] as illustrated in Fig.1. To model the overlay knowledge two components uncertainty management approach which is used for this purpose.

**Overlay knowledge model with Bayesian Networks:** The Bayesian Network is a popular representation for encoding uncertain expert knowledge [13]. A Bayesian network is an acyclic directed acyclic graph (DAG) where each node E represents a domain variable and each arc V between nodes represents a probabilistic dependency, quantified using a conditional probability distribution.

General case of Bayesian networks consists of a set of n nodes X={X1, X2, ..., Xn} organized in a DAG, where each node Xi has parents pa(Xi), the joint probability distribution is compactly expressed as

\[ p(x_1, \ldots, x_n) = \prod_{i=1}^{n} p(x_i \mid pa(x_i)) \]  

Bayesian networks most often represent causal statements of the kind \( X \rightarrow Y \), where X is a cause of effect Y. Reverend Thomas Bayes (1702–1761) provided the famous Bayes’ rule:

\[ P(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)} = \frac{P(X,Y)}{P(Y)} \]  

Note also:

\[ P(Y) = \sum_x P(Y \mid X)P(X) = \sum_x P(X,Y) \]  

Bayesian Networks provide two important capabilities: diagnosis (inferences about possible
causes of an event) and prediction (future state/evolution of variables given evidence). Learner modeling is a domain in which there are many different sources of uncertainty and/or imprecision, therefore numerically approximate reasoning techniques like Bayesian networks is suitable for this purpose [14, 15, 16].

In a learner model based on Bayesian Network, internal nodes represent the concepts of the knowledge domain model and the leaf nodes represent the evidences. The sources of evidence are the results of learner interaction with the system (e.g. answers to questions or exercises, time spent reading certain content, number of clicks, etc.).

C. Open Learner Modeling

An Open Learner Model makes a machine’s representation of the learner available as an important means of support for learning. This means that a suitable interface is created for use by learners, and in some cases for others who aid their learning, including peers, parents and teachers [3].

Opening the learner model generally involves more than simply showing the learner the representations from the underlying system's model of their knowledge (or other attributes modelled), as these representations are not usually designed for interpretation by humans. Generally the presentation could be made in a graphical or textual way. The purpose of OLM is not dependent on a graphical or textual representation. In fact, the important matter is the comprehensibility and clarity of OLM presentation. The open learner model can be commonly represented in two main categories: simple and complicated [17]. In this paper the skill meter is used, which is the most common way of OLM presentation. As [18] considered, skill meter is a part-shaded bar showing learner progress as a subset of expert knowledge, or the probability that a learner knows a concept.

D. Related Works

One of the main purposes of Open Learner Modeling is improving accuracy of learner model. To accomplish this aim learner model can change according to the idea of learner about his/her learner model. Related works, which focused on this aim, are Mr. Collin [5], STyLE-OLM [6], and ChatBot [7].

In these systems first, learner model is presented to learner then learner can inspect his/her model and negotiate with system. To update learner model according to learner’s belief, two separate belief or confidence measures are considered:

- The first one reflects the learner's own belief about his/her performance.
- The second one is the system's evaluation of the learner’s performance. For updating the learner model the equation (4) is used.

\[ U = |L - S| \]  

(4)

In equation (4) Update Measure value is shown by U, L stands for learner confidence and S stands for system confidence, also Update Measure is computed with the measure of difference between the system confidence and the learner confidence. Learner and system confidence values are expressed in four levels: 1- very sure, 2- almost sure, 3- unsure and 4- completely unsure. If Update Measure is less than or equal to one, the system trusts in learner’s belief. But if trust value is more than one, the system negotiates with the learner for some correction or modification in learner model. For example in such a situation the system supplies another exam and the learner is requested to answer the questions. If the learner correctly answers to the questions, the system will trust to his/her feedback in the process of interaction. Otherwise the system does not trust to learner’s response and learner model is remained unchanged.

For instance in table (1) three learner confidence, system confidence and Update Measure for learners a, b and c is shown, also according to Update Measure is determined which user is trustworthy and which one isn’t. Update Measure of a is 1 and b is 0 as Update Measure of both a and b is less equal to 1 they are trustworthy and the model update according to their belief. Update Measure of c is more than 1 so c is not trustworthy so the model won’t updated.

<table>
<thead>
<tr>
<th>Learner Name</th>
<th>learner confidence (L)</th>
<th>system confidence (S)</th>
<th>Update Measure (U)</th>
<th>Should Update? (Yes/No)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>No</td>
</tr>
<tr>
<td>c</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>Yes</td>
</tr>
</tbody>
</table>

III. METHODOLOGY

As mentioned, accuracy of learner model is an important issue. One approach which is used to improve learner model accuracy is open learner modeling. In this paper a novel method is proposed to improve accuracy of learner model based on learner knowledge and learner belief. For this purpose a research plan is arranged (Fig.2). This plan has five steps including: 1) selecting participants and learning concepts, 2) learner modeling, 3) open learner modeling, 4) updating learner model with proposed method, and 5) analyzing the results. Each step will be explained in following subsections.

A. SELECTING PARTICIPANTS AND LEARNING CONTENTS

Participants characteristic: In this survey, 16 students of Payame Noor University with an average age of 20 participated. They were 6 computer
engineering students, 7 Information Technology students, and 3 math students.

Learning Contents: A part of the Data Structure and Algorithms lesson were selected. The selected contents include:
- C1 = “Add and Delete Node in Link List”
- C2 = “Scan and Use of Link List”
- C3 = “Time Order”
- C4 = “Push and Pop and Use of Stack”

As mentioned, in this paper an overlay knowledge modeling approach with Bayesian Network is used. Fig.3a shows knowledge model of learner in which internal nodes represent the concepts of domain and the leaf nodes represent the questions (q1, q2, …, q12). Each question is about a learning content. For example, as shown in fig 3a, q1, q2, and q3 are related to concept C1.

\[
P(q_1|C_1) = P(q_1=1|C_1=1)\cdot P(C_1=1) + P(q_1=1|C_1=0)\cdot P(C_1=0)
\]

\[
P(q_2|C_1) = P(q_2=1|C_1=1)\cdot P(C_1=1) + P(q_2=1|C_1=0)\cdot P(C_1=0)
\]

\[
P(q_3|C_1) = P(q_3=1|C_1=1)\cdot P(C_1=1) + P(q_3=1|C_1=0)\cdot P(C_1=0)
\]

\[
P(q_1|C_2) = P(q_1=1|C_2=1)\cdot P(C_2=1) + P(q_1=1|C_2=0)\cdot P(C_2=0)
\]

\[
P(q_2|C_2) = P(q_2=1|C_2=1)\cdot P(C_2=1) + P(q_2=1|C_2=0)\cdot P(C_2=0)
\]

\[
P(q_3|C_2) = P(q_3=1|C_2=1)\cdot P(C_2=1) + P(q_3=1|C_2=0)\cdot P(C_2=0)
\]

\[
P(q_6|C_2) = P(q_6=1|C_2=1)\cdot P(C_2=1) + P(q_6=1|C_2=0)\cdot P(C_2=0)
\]

\[
P(q_9|C_3) = P(q_9=1|C_3=1)\cdot P(C_3=1) + P(q_9=1|C_3=0)\cdot P(C_3=0)
\]

\[
P(q_7|C_3) = P(q_7=1|C_3=1)\cdot P(C_3=1) + P(q_7=1|C_3=0)\cdot P(C_3=0)
\]

\[
P(q_11|C_4) = P(q_11=1|C_4=1)\cdot P(C_4=1) + P(q_11=1|C_4=0)\cdot P(C_4=0)
\]

\[
P(q_12|C_4) = P(q_12=1|C_4=1)\cdot P(C_4=1) + P(q_12=1|C_4=0)\cdot P(C_4=0)
\]

The knowledge of learner regarding his answers is calculated as follows:

\[
k_i = P(C_1=1|q_1=1) \cdot P(q_2=1|C_1=1) \cdot P(q_3=1|C_1=1)
\]

The same calculation is used for C2, C3, and C4.

As mentioned, we use k1, …, k4 as value of learner’s knowledge in concepts C1, …, C4. And because these are probability values, they will be a number between 0 and 1.
Finally, as C1, C2, C3, and C4 are independent; \( k_1 \times k_2 \times k_3 \times k_4 \) is used to compute the whole learner’s knowledge.

C. OPEN LEARNER MODELING

After modeling learner’s knowledge, the model was shown to learners as Fig.4. The first column shows the concepts, and the second column shows the value of \( k_1, k_2, k_3, \) and \( k_4 \), numerically and also as skill meter.

After observing the measured knowledge’s value in individual concepts, each learner could express his/her belief about each concept’s value in the last column. S/he may agree with the calculated knowledge’s value or believe that his/her knowledge in each concept is more or less than what is shown. For example Fig.4 shows 0.257 (=\( k_1 \)) as a learner’s knowledge in concept C1. Suppose the learner believes that his/her knowledge in concept C1 is a number like 0.7, so s/he could enter the new value in third column. The learner’s belief about different concepts should be entered as a number between 0 and 1.

Actually, updating is based on the learner belief and the calculated value of knowledge in the previous step. Using this update, we aim to increase the accuracy of the calculated value of learner’s knowledge and in fact, learner model.

**Taking a Comprehensive Test:** After updating learner model by means of proposed method (formula 5), we tried to evaluate the accuracy of updated learner model. For this purpose, we took a comprehensive test and compared the knowledge value that was obtained by this test and knowledge value that was obtained by formula 5. The comprehensive test includes 10 questions that each question is related to a Ci. We used Ti to refer to knowledge value in concept Ci that has been obtained by comprehensive test.

E. ANALYZING THE RESULTS

We are going to investigate whether the proposed method and updating model by using learner’s belief, has improved the accuracy of knowledge model. For this purpose we studied that the amount of which of knowledge (\( K_i ' \) or \( K_i \)) is closer to result of the comprehensive test. As mentioned \( K_i \) and \( K_i ' \) are the values of learner’s knowledge in Ci before and after updating by formula 5.

At first the \( T_i - K_i ' \) and \( T_i - K_i \) were computed. If the difference between \( T_i \) and \( K_i ' \) is less than the difference between \( T_i \) and \( K_i \), it seems that our proposed method has improved the accuracy of learner model. We applied student’s t-test (t-test) to study whether the result is meaningful. In following subsections t-test and results are explained.

**T-test (Student’s t-test):** Student’s t-test, in statistics, is a method of testing hypotheses about the mean of a small sample drawn from a normally distributed population when the population standard deviation is unknown [19]. In simple terms, the t-test compares the actual difference between two means about the variation in the data. There are three types of t-test:

- **One-sample t-test:** Used to compare a sample mean with a known population mean or some other meaningful, fixed value. That is subjects are randomly drawn from a population and the distribution of the mean being tested is normal.

- **Independent samples t-test:** Used to compare two means from independent groups.

- **Paired samples t-test:** This is appropriate for testing the mean difference between paired observations when the paired differences follow a normal distribution. It uses when the observed data are from the same subject and we are going to compare data before-after a situation [20, 21].

We are going to compare accuracy of knowledge model before and after use of proposed method (formula 5), hence we have the same subject (knowledge model) and we should use the Paired samples t-test.

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**Fig. 4. Opening model of learner**

<table>
<thead>
<tr>
<th>Concepts:</th>
<th>Your Knowledge in System</th>
<th>Your Idea About Your Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add and Delete in Link List</td>
<td>0.257</td>
<td></td>
</tr>
<tr>
<td>Scan and Use of Link List</td>
<td>0.821</td>
<td></td>
</tr>
<tr>
<td>Time Order</td>
<td>0.312</td>
<td></td>
</tr>
<tr>
<td>Push and Pop and Use of Stack</td>
<td>0.732</td>
<td></td>
</tr>
</tbody>
</table>

(5)

\[
k_i' = k_i + \frac{(L_B_i - k_i)}{k_i + 1}
\]
As mentioned, we aim to use Paired \( t \)-test to show that the proposed method has improved the accuracy of learner model. In other words, we should show that \( K_i' \) to compare \( K_i \) is closer to \( T_i \) or \( T_i-K_i' \) is less than \( T_i-K_i \). And we should also show there is meaningful difference between the means of \( T_i-K_i' \) and \( T_i-K_i \). In order to apply \( t \)-test, the variables should be normal. We used MiniTab for normality test as well \( t \)-test.

**Normality test in MiniTab:** As stated, we aimed to compare \( T_i-K_i \) to \( T_i-K_i' \) \((i \in \{1, 2, 3, 4\})\). At first, normality of each \( T_i-K_i \) and \( T_i-K_i' \) was assessed using MiniTab. For example Fig.5 shows normality test for each \( T_i-K_i \) using MINITAB.

MINITAB generates a normal probability plot and performs a hypothesis test to examine whether the observations follow a normal distribution or not. For this test, the hypotheses are:

\[ H_0 : \text{data follow a normal distribution} \]
\[ H_1 : \text{data do not follow a normal distribution} \]

The null hypothesis would be rejected as the \( p \)-value is less than alpha \((\alpha)\) level, and it means the data is highly non-normal. Note that the \( p \)-value is the probability of incorrectly rejecting the null hypothesis.

The alpha is the maximum acceptable level of risk for rejecting a true null hypothesis and is expressed as a probability ranging between 0 and 1. The most commonly used alpha level is 0.05 [15] and in this paper the value of 0.05 is considered as the alpha level.

**The results**

Based on results, the null hypothesis would be rejected if \( p \)-value became less than alpha level. We would fail to reject the null hypothesis if \( p \)-value was greater than alpha level and it would conclude that there is meaningful difference in the mean of the two groups [16].

For example, Fig.6(a) states the result of Paired \( t \)-test using MINITAB, for \( T_i-K_i' \) and \( T_i-K_i \). The figure shows that the mean value of \( T_i-K_i' \) is -0.095 that is less than the mean value of \( T_i-K_i \) (0.470). So it seems proposed method has improved the accuracy of knowledge model. Also the computed \( p \)-value is 0.004, which is less than considered alpha value. This small \( p \)-value suggests that the data are inconsistent with \( H_0 \) and \( H_0 \) is rejected. Therefore the \( H_1 \) is confirmed and there is a meaningful difference between two means. So the proposed method has improved the accuracy of knowledge model in concept C1.

We also used T-test for comparing \( T_2-K_2 \) to \( T_2-K_2' \), \( T_3-K_3 \) to \( T_3-K_3' \), and \( T_4-K_4 \) to \( T_4-K_4' \). The results of comparing have shown in Fig.6 (b, c, d). A summarized of T-test results, is shown in Table3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean Value</th>
<th>( p )-value From Normality Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_i-K_i' )</td>
<td>0.095</td>
<td>0.332 greater than 0.05</td>
</tr>
<tr>
<td>( T_i-K_i )</td>
<td>0.470</td>
<td>0.193 greater than 0.05</td>
</tr>
<tr>
<td>( T_2-K_i' )</td>
<td>0.324</td>
<td>0.456 greater than 0.05</td>
</tr>
<tr>
<td>( T_2-K_i )</td>
<td>0.552</td>
<td>0.295 greater than 0.05</td>
</tr>
<tr>
<td>( T_3-K_i' )</td>
<td>0.405</td>
<td>0.093 greater than 0.05</td>
</tr>
<tr>
<td>( T_3-K_i )</td>
<td>0.492</td>
<td>0.316 greater than 0.05</td>
</tr>
<tr>
<td>( T_4-K_i' )</td>
<td>0.313</td>
<td>0.213 greater than 0.05</td>
</tr>
<tr>
<td>( T_4-K_i )</td>
<td>0.520</td>
<td>0.101 greater than 0.05</td>
</tr>
</tbody>
</table>

Table3. The Result of T-test For \( T_i-K_i' \) and \( T_i-K_i \)

**Paired samples \( t \)-test in MiniTab:** For a paired \( t \)-test:

\[ H_0 : \mu_2 = \mu_1 \]
\[ H_1 : \mu_2 \neq \mu_1. \]

Where \( \mu_2 \) and \( \mu_1 \) are mean of \( T_i-K_i' \) and \( T_i-K_i \). In other words, \( H_0 \) expresses there is no meaningful difference between the means of \( T_i-K_i' \) and \( T_i-K_i \); and \( H_1 \) states the difference is significance.

**Table2. The Result of Normality Test For \( T_i-K_i' \) and \( T_i-K_i \)**

<table>
<thead>
<tr>
<th>Variables</th>
<th>P-value of Paired t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_i-K_i' ) and ( T_i-K_i )</td>
<td>0.004 less than 0.05</td>
</tr>
<tr>
<td>( T_2-K_i' ) and ( T_2-K_i )</td>
<td>0.002 less than 0.05</td>
</tr>
<tr>
<td>( T_3-K_i' ) and ( T_3-K_i )</td>
<td>0.017 less than 0.05</td>
</tr>
<tr>
<td>( T_4-K_i' ) and ( T_4-K_i )</td>
<td>0.063 greater than 0.05</td>
</tr>
</tbody>
</table>

The \( p \)-value of Paired t-test for \( (T_2-K_2' \) and \( T_2-K_2) \) and \( (T_3-K_3' \) and \( T_3-K_3) \) are less than alpha value, hence the data are inconsistent with \( H_0 \) and \( H_0 \) is rejected. Therefore the \( H_1 \) is confirmed and there are meaningful differences between two means. As a
result, our proposed method has improved accuracy of learner model in two concepts C2 and C3 same as C1.

The p-value of Paired t-test for \((T_4-K_4)\) is 0.063 that is greater than considered alpha level (0.05). Consequently, we cannot reject the null hypothesis. It means we could not confirm that proposed method could improve the accuracy of knowledge model in concept C4. But if we consider 0.1 as alpha level (it means we accept more risk to reject a true null hypothesis) the value 0.063 is less than 0.1 and the null hypothesis \((H_0 of \text{Paired t-test})\) will be rejected. Therefore our proposed method would improve the accuracy of knowledge model in concept C4.

In this survey, in addition of C1, C2, C3, and C4, another concept had been considered, but since the values of \(T_i-K_i\)' for this concept was non-normal, we could not rely on the Paired t-test of this concept. So we ignored this concept.

IV. CONCLUSION

As mentioned the accuracy of learner model is an important issue. Open learner modeling is an approach which is used for this aim. In this paper a novel method was proposed to improve accuracy of learner model based on learner knowledge and learner belief. For this purpose the knowledge model of 16 students in 4 concepts of Data Structure and Algorithms lesson, were obtained and represented by use of the overlay learner modeling using Bayesian Networks. Then we presented the learner model as skill meter and learner stated his/her belief about it. Then the model was updated through a proposed method. At last we took a comprehensive test to evaluate our method. Since we were going to compare the knowledge model before and after applying our proposed method, we used Paired t-test. For this idea we applied Paired t-test in MINITAB to compare the mean of difference between comprehensive test and updated knowledge model (computed by proposed method) on the one hand and the mean of difference between comprehensive test and knowledge model (before update) on the other hand. The obtained results have shown that for three concepts the p-value of Paired t-test is less than 0.05(alpha value=0.05) and for one concept the p-value is less than 0.1(alpha value=0.1). It means for selected concepts our method have improved the accuracy of learner model.

As future works, we are going to develop our method by following capacity:

To make use of other parameters of learner model: As mentioned, learner model has several parameters such as emotions, background, and individual traits. In this study we used knowledge model. We are going to apply other parameters to improve our method.

To negotiate with learner about his/her belief on his/her learner model: It means when learner expresses his/her idea about his/her learner model, system negotiates with the learner to assess the truth of learner idea, and then learner model is updated. To assess accuracy of learner belief, system could use other questions about related concept, and then based on answer decides whether update learner model or not.

To follow a line of investigation about learning contents: According to table 2, the difference between mean of \(T_i-K_i\) and \(T_i-K_i\) in variant concepts is different. The third concept \((C_3)\) is mathematical, and it has minimum difference mean of \(T_3-K_3\) and \(T_3-K_3\). So the type of concept could be considered as another parameter of proposed method.
Normality Test for $T_1 - K_1'$ (P-value= 0.332)

Normality Test for $T_1 - K_1$ (P-value= 0.193)

Normality Test for $T_2 - K_2'$ (P-value= 0.456)

Normality Test for $T_2 - K_2$ (P-value= 0.295)

Normality Test for $T_3 - K_3'$ (P-value= 0.093)

Normality Test for $T_3 - K_3$ (P-value= 0.316)

Normality Test for $T_4 - K_4'$ (P-value= 0.213)

Normality Test for $T_4 - K_4$ (P-value= 0.101)

Fig. 5. Normality Test in MINITAB
Paired T for = T1-K1' - T1-K1

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-K1</td>
<td>16</td>
<td>-0.095</td>
<td>0.562</td>
<td>0.141</td>
</tr>
<tr>
<td>T1-K1'</td>
<td>16</td>
<td>0.470</td>
<td>0.300</td>
<td>0.075</td>
</tr>
<tr>
<td>Difference</td>
<td>16</td>
<td>-0.565</td>
<td>0.738</td>
<td>0.184</td>
</tr>
</tbody>
</table>

95% upper bound for mean difference: -0.241
T-Test of mean difference = 0 (vs < 0): T-Value = -3.06  P-Value = 0.004

(a) T-test Result for T1-K1' and T1-K1

Paired T for = T2-K2' - T2-K2

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>T2-K2</td>
<td>16</td>
<td>0.3237</td>
<td>0.2386</td>
<td>0.0597</td>
</tr>
<tr>
<td>T2-K2'</td>
<td>16</td>
<td>0.5521</td>
<td>0.2505</td>
<td>0.0626</td>
</tr>
<tr>
<td>Difference</td>
<td>16</td>
<td>-0.2283</td>
<td>0.2765</td>
<td>0.0591</td>
</tr>
</tbody>
</table>

95% upper bound for mean difference: -0.1071
T-Test of mean difference = 0 (vs < 0): T-Value = -3.30  P-Value = 0.002

(b) T-test Result for T2-K2' and T2-K2

Paired T for = T3-K3' - T3-K3

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>T3-K3</td>
<td>16</td>
<td>0.4045</td>
<td>0.1925</td>
<td>0.0481</td>
</tr>
<tr>
<td>T3-K3'</td>
<td>16</td>
<td>0.4920</td>
<td>0.1766</td>
<td>0.0442</td>
</tr>
<tr>
<td>Difference</td>
<td>16</td>
<td>-0.0875</td>
<td>0.1505</td>
<td>0.0376</td>
</tr>
</tbody>
</table>

95% upper bound for mean difference: -0.0215
T-Test of mean difference = 0 (vs < 0): T-Value = -2.33  P-Value = 0.017

(c) T-test Result for T3-K3' and T3-K3

Paired T for = T4-K4' - T4-K4

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>T4-K4</td>
<td>16</td>
<td>0.5209</td>
<td>0.3161</td>
<td>0.0780</td>
</tr>
<tr>
<td>T4-K4'</td>
<td>16</td>
<td>0.3130</td>
<td>0.2982</td>
<td>0.0745</td>
</tr>
<tr>
<td>Difference</td>
<td>16</td>
<td>0.208</td>
<td>0.414</td>
<td>0.104</td>
</tr>
</tbody>
</table>

95% CI for mean difference: (-0.013, 0.429)
T-Test of mean difference = 0 (vs not = 0): T-Value = 2.01  P-Value = 0.063

(d) T-test Result for T4-K4' and T4-K4

Fig. 6. T-test Result for T1-K1' and T1-K1
REFERENCES


